TRANSIENT STABILITY ANALYSIS OF POWER SYSTEMS USING LIAPUNOV FUNCTIONS WITH IMPROVED SYNCHRONOUS MACHINE MODELS

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OCTOBER, 1973

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY

By
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to the

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

OCTOBER, 1973

CERTIFICATE

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Dated: 8/4/74

ACKNOWLEDGEMENTS

It is with greatest pleasure that I acknowledge my deep gratitude to Dr. M.A. Pai who suggested the problem and provided excellent guidance throughout the course of this work.

I am thankful to my fellow-research scholars M/s P.G. Murthy, M.A. Khan, and K.S. Prakasa Rao (Department of Electrical Engineering)and Dr. Raghavendra,V., (Department of Mathematics) for the many interesting discussions on related areas.

I take this opportunity to express my gratitude to Principal B.H. Karakaraddi and the Board of Governors of Karnataka Regional Engineering College, Suratkal, for having sponsored and deputed me for this work under the Q.I.P. scheme.

My deep appreciation goes to Dr. K.R. Sarma for his friendly advice on many of my decisions related to this program.

I am happy to acknowledge the facilities for research provided by the authorities of I.I.T.Kanpur. Finally I thank Mr. K.N. Tewari for his patient and skilful typing.

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H	Inertia constant of the machine ; MJ/MVA
M	Inertia constant; p.u. power sec.^{2}/rad.
D	Damping constant ; p.u. power sec.^{2}/rad.
x_d	Direct axis synchronous reactance
x_q	Quadrature axis synchronous reactance
x_d'	Direct axis transient reactance
x_q'	Quadrature axis transient reactance
T_o'	Direct axis transient open circuit time constant
diag	Diagonal matrix
col	Column matrix

A dot over a symbol indicates a differentiation with respect to time.

A bar below a symbol signifies a vector.

A superscript 'T' denotes 'transpose'.

$X_{ii} = 1/Y_{ii}$

$X_{ij} = 1/Y_{ij}$

SYNOPSIS

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October 1973
TRANSIENT STABILITY ANALYSIS OF POWER SYSTEMS USING LIAPUNOV FUNCTIONS WITH IMPROVED SYNCHRONOUS MACHINE MODELS

The transient stability problem arises in a power system when the power balance between the mechanical input and electrical output of synchronous machines gets altered suddenly due to occurence of large disturbances such as faults, removal or addition of large loads etc. Depending on the nature and location of the disturbance, the rotors of the machines may accelerate or decelerate. If the system settles down to a steady-state following such a disturbance or after the fault has been cleared as the case may be, the system is said to be stable. Historically this problem has been studied by the step-by-step methods which is still the widely used method with the aid of the digital computer.

In recent years however, the Liapunov's direct method has been applied to investigate the transient

stability problem and the method is now acknowledged as being able to give results in satisfactory concordance with those provided by the step-by-step method. Furthermore the method has certain advantages. When the transient state has occured due to a fault, an important piece of information of interest is the critical fault-clearing time. The Liapunov function approach yields this information through a single integration of the faulted system differential equations while in the step-by-step method, repetetive integration of the system equations for different assumed clearing times is involved, each integration necessarily having to be carried on far beyond the fault clearing instant, even for the'first swing' stability investigation. For a given clearing time the method will also indicate whether the system will remain stable or not. The Liapunov method can also be used to specify stability margins or stability indices, without making any actual-stability calculations. Because of all these reasons, the chances are, that Liapunov approach may prove to be an effective method for on-line-stability and security analysis. However, there are certain difficulties still to be overcome with the method. A chief drawback of the Liapunov approach has been the conservative nature of the results. One of the contributing factors is inherent

in the method itself, namely the sufficiency nature of the Liapunov's theorem. Another important factor has been the simplified- and therefore approximate - models of the synchronous machines that have been used. It is well-known that factors like damping, saliency, transfer conductance, flux-decay, voltage regulator and governor dynamics have appreciable effects on the stability of a power system. However, inclusion of all these factors in modelling the synchronous machine results in very complex system of equations and the derivation of a Liapunov function becomes quite difficult and in some cases impossible as of to-day, particularly when dealing with multimachine systems. In the literature, many of these effects have not been taken into account so far in a comprehensive manner. This thesis seeks to cover some new grounds in this direction. The power system problem is formulated with the synchronous machine modelled with varying degree of details and Liapunov function is sought for each case. Results are obtained both for single machine and multimachine systems.

The basic philosophy in general, is to cast the power system dynamics in the form of an 'indirect control system' equations of Lur'e's form and then derive a Liapunov function. Depending upon the nature and form of the nonlinearities, different techniques

are called for, in arriving at a Liapunov function. Once the Liapunov function is available, the stability region or equivalently the critical fault-clearing time can be computed. The effect of parameter variation can also be studied with great ease and speed, using this Liapunov function. A brief outline of the various chapters is given below.

The first chapter is devoted to the statement of the problem of transient stability in power systems using the Liapunov method. The scope and objectives of the thesis are enumerated and organization of the thesis material is briefly sketched.

The second chapter deals with the transient stability problem including saliency effects. A single machine connected to an infinite bus is considered first, with the synchronous machine modelled with damping and transient saliency taken into account. A Liapunov function of the 'quadratic form plus integral of nonlinearity' type is developed using Kalman's construction procedure. This Liapunov function is used to determine the stability region and the critical clearing time. A detailed parameter analysis is carried out for a specific numerical example and useful conclusions drawn. The results with and without transient saliency are compared. Next a two-machine

power system with transient saliency, is dealt with. Two distinct cases arise when dealing with multimachine systems; one is known as a system with 'uniform damping' while the other as a system with 'nonuniform damping'. Both these cases are discussed and appropriate Liapunov functions derived. A general k-machine system with saliency is briefly discussed. The difficulty in systematically arriving at a Liapunov function for this system is pointed out. It is due to the fact that when cast in the Lur'e form, some of the terms in the nonlinearities cannot be determined explicitly in terms of the state variables.

The third chapter considers a single machine-infinite bus system. The synchronous machine is at first modelled with the flux-decay and voltage regulator dynamics included in addition to damping. This model results in two nonlinearities requiring a matrix version of Kalman's construction procedure. However, a direct application of this procedure is difficult because of the fact that the nonlinearities happen to be of the 'multi-argument' type. Using results of Desoer and Wu on stability of nonlinear systems with multiplicative nonlinearities, a systematic method is evolved to arrive at a Lyapunov function having the required properties in the region of interest. Effects of voltage regulator

parameters and flux decay on the critical clearing time are studied through a numerical example. Next a more general model of the synchronous machine including transient saliency, flux decay, voltage regulator and governor dynamics, is used to derive a generalized Liapunov function.

Chapter four is concerned with multimachine system with flux decay effect included. It is observed that the resulting model is not amenable to the procedures discussed so far. Hence, recourse is taken to a version of 'integration by parts' method to arrive at a Liapunov function. Using this technique Liapunov functions are derived for two-machine and three-machine systems. The results of the two-machine and three-machine systems are extended to a general k-machine system, by induction, to arrive at an appropriate Liapunov function.

Chapter five deals with power systems with transfer conductances. A two-machine system is formulated and a Liapunov function derived using Kalman's construction procedure for the uniformly damped case. A three-machine formulation on similar lines reveals that the Popov frequency criterion is not satisfied for this system. Indeed this is true for systems having more than three machines also. Hence this particular line of attack has to be upto two-machine system.

In chapter six a brief review of the results and some concluding remarks are given. The problems that have surfaced during the course of this work and which require further investigation are indicated.

CHAPTER I

INTRODUCTION

1.1 STABILITY OF POWER SYSTEMS

'Power system stability' is a term applied to alternating current power systems, denoting a condition in which the various synchronous machines in the system remain in synchronism or 'in step' with each other. Conversely 'instability' denotes a condition involving loss of synchronism, or falling 'out of step'[1]. The study of stability for purposes of analysis, has been divided into two major divisions:

(i) 'Steady state stability' which is concerned with the stability under small perturbations or small variations around an operating point.

(ii) 'Transient stability' which is concerned with the stability under severe disturbances such as a fault or sudden load changes or sudden changes in line reactances due to switching operations. These disturbances may be termed as large perturbations.

Although stability is one phenomena, the above division has been made primarily as an aid to analysis[2]. In fact the above two are the terminal phases of a single problem. For example, a test for the system for an assumed fault consists of two steps: A check as to whether the

system is 'steady state stable' after the fault has been removed or isolated and the oscillations have died out, is first made. If it is 'steady state stable', then it is tested for transient stability, generally by what is known as the 'first swing stability'. This consists in merely observing the swing curves, which are the plot of the rotor angles versus time, for the duration of the first swing and checking whether the relative rotor angles diverge or attain a steady state. If the rotor angles appear to diverge, the system is considered to be transiently unstable. There exists an intermediate period following the first swing and until the stable steady state condition has been reached. During this subsequent period, the control devices are active and a study of the system behaviour during this period should consider the dynamics of these control devices. Such a study falls under the scope of 'dynamic stability'. In most cases the system remains stable in the subsequent swings if it is stable during the first swing. Therefore it is usual to consider a system as stable if it is 'steady state stable' and 'first swing stable' under transient state. A rare exception can however occur owing to dynamic instability due perhaps to abnormally slow exciter response or inappropriate adjustment of automatic voltage regulators[3]. The steady state stability investigation is straightforward and relatively simple. The determination of

the transient stability however, is more involved. In the most general form, it involves the combination of a solution of nonlinear algebraic equations describing the network and a solution of the nonlinear differential equations.

1.2 TRANSIENT STABILITY PROBLEM

Transient stability can be defined as follows:

'An electrical power system is said to be in a transient stability state with respect to a large perturbation if, after such a perturbation, it recovers a synchronous operating state'. (Conference Internationale des Grands Réseaux Electriques a Haute Tension, 'CIGRE', Paris, 8-18 June, 1966).

The large perturbation which creates the stability problem, may be a sudden increase in load, or a sudden increase in reactance of the circuit, caused for example by a line outage, or by a disconnection of one of two or more parallel lines as a normal switching operation. But the most severe perturbation to which a power system can be subjected to, is a short circuit, generally termed as 'fault'. Although unsymmetrical faults like one-line to ground, line to line or two-line to ground faults are more common in practice, a symmetrical three phase short circuit is the most severe of all the faults. For example a three phase short circuit on a line connecting a generator and a motor entirely cuts off the flow of power between the

machines and results in the acceleration of the generator and deceleration of the motor and an eventual loss of synchronism unless the short circuit is quickly removed. A similar situation arises in a multimachine system wherein one or more machines may experience acceleration or deceleration following such a fault resulting in loss of synchronism if the fault is not cleared. On the other hand, if the fault is an unsymmetrical fault involving only one or two lines, then some synchronizing power can still be transmitted past the fault. In such a situation, under certain load conditions, the system may remain stable even with a sustained fault. However, a system has to be planned and designed for all possible contingencies including the worst conditions. Thus in all stability studies, a three phase short circuit must be considered[1].

Granting therefore, that the most severe type of disturbance or perturbation is a short circuit, the transient stability problem may be formulated for this eventuality in the following manner:

Given a power system initially in steady state operation, assume that a fault occurs at time t_o which can be taken as zero without loss of generality. Two questions arise:

(i) Is there a stable equilibrium position to which the system settles after the fault is cleared?

(ii) If the answer to (i) is 'yes', what is the maximum time that the fault may be allowed to remain so that the system will settle to the stable equilibrium point after the fault is cleared? This is termed as the critical fault-clearing time or simply as critical clearing time.

There are two methods to solve the above problem:

1. The simulation method: This is presently the most widely used approach and studies the problem through analog simulation or by some numerical step-by-step integration method. Historically the network analyzer was being used extensively for quite sometimes. The capacity of the network analyzer has been limited however, thereby restricting its use for reasonably small systems having limited number of generators. But with the growth of large power systems due to addition of generators and increasing interconnections of various existing systems, power system engineers have changed over to the use of digital computers for stability analysis by numerical method. The standard numerical integration method approach divides the transient stability problem into two phases:

Phase A: This phase consists the study of the evolution of the system from the instant of fault to an assumed clearing time t_e.

Phase B: This involves the study of the evolution of the system in its post-fault period.

The model can have detailed generator representation to include flux decay, voltage regulator, governor action etc. For an assumed clearing time t_e, the transient stability is examined. Clearly, if t_e is less than the critical clearing time t_c, the system will remain stable. In general however, the critical clearing time t_c will not be known apriori. It is determined by several trials of assumed values of clearing time t_e. Thus the estimation of t_c involves repetitive computations because of phase B and hence time consuming.

2. 'Direct analysis' method: By this method, the stability is investigated without actually solving the differential equations. The equal area criterion[1,2], the phase-plane method[4], the energy integral method[5,6] and now the Liapunov's direct method[7-28] fall under this category. While the first two of the above are applicable only for second order systems, the last two methods can be used for higher order systems and thus for multimachine systems.

1.3 LIAPUNOV'S DIRECT METHOD

The Liapunov method is more general than the energy integral method. In fact, energy integral happens to be a particular case of a specific Liapunov

function, viz. when the total system energy is chosen as the Liapunov function. But in general, Liapunov function will not be restricted to the energy function. Indeed, the more accurate Liapunov functions which yield larger stability regions, are not exactly the energy integrals of the system. It can therefore be safely speculated that among all direct methods, Liapunov approach is the only method that can possibly compete with the standard step-by-step method. There are good reasons to visualize such a prospect as discussed below.

The Liapunov's direct method eliminates the phase B computation discussed in method 1 above. Instead, a stability criterion is used. This criterion is based on the construction of a Liapunov function V for phase B system around the post-fault equilibrium point and its comparison with a limit value C.

Let the dynamic equations for the faulted and post-fault condition be denoted respectively by

$$\dot{\underline{x}} = F_1(\underline{x}) \tag{A}$$

$$\dot{\underline{x}} = F_2(\underline{x}) \tag{B}$$

At the outset, a Liapunov function $V(\underline{x})$ is constructed for system (B) and a constant C is computed such that $V(\underline{x}) < C$ constitutes a region of attraction around the post-fault stable equilibrium point. The constant C is generally found by evaluating the function $V(\underline{x})$ for

system (B) at the unstable equilibrium point nearest to the stable post-fault equilibrium point. Now, given a clearing time t_e, the faulted system (A) is numerically integrated and the value of the state vector $\underline{x}$ is computed at the end of t_e. Now with this value of the state vector, the Liapunov function V is evaluated. This value of V is compared with C. The system is considered stable if $V \leq C$. The critical clearing time also can be estimated by evaluating the V-function at the end of each integration step and comparing it with C. That value of t at which $V(\underline{x}) = C$ gives the critical clearing time t_c. The computation of t_c thus involves just one integration of system (A) and does not require any integration of system (B) unlike in the method 1. The saving in computation time can be significant, particularly if a large system is to be examined for its transient stability property for various contingencies. Here the role of the Liapunov method is to be considered as complementing rather than supplementing the actual detailed simulation. A preliminary screening by Liapunov method may reduce the number of studies to be done in detail. Further, the influence of the various parameters can be thoroughly investigated with great ease and speed. Another advantage of the Liapunov method is that it can be used to specify stability margins or stability indices without making actual-stability

calculations. Gradually it is being acknowledged that the Liapunov method is able to give results in satisfactory concordance with those provided by the step-by-step method. The indications at present are, that this approach may still succeed in being accepted as an effective tool for on-line-stability and security analysis. However, there are certain difficulties still to be overcome with this method.

A chief drawback of the Liapunov method has been the conservative nature of the results. One of the contributing factors is inherent in the method itself, namely the sufficiency nature of the Liapunov theorem. Another important factor has been the simplified- and therefore approximate-models of the synchronous machines that have been used. It is well-known that factors like damping, transient saliency, transfer conductance, flux decay, voltage regulator and governor dynamics, have appreciable effects on the stability of a power system. But, inclusion of all these factors in modelling the synchronous machine results in a set of system equations which are highly complex and the derivation of a reasonably good Liapunov function becomes quite difficult and in some cases impossible as of to-day. This is more so in multimachine system. Many of these effects have not been taken into account in a comprehensive manner in the literature while applying the Liapunov's method.

This thesis seeks to cover some new grounds in this direction. The power system problem is formulated with the synchronous machine modelled with varying degree of details and a Liapunov function sought for each case. Both single and multimachine systems are considered. Before giving an outline of thesis, some preliminaries are discussed.

1.4 THE CONSTRUCTION OF LIAPUNOV FUNCTION

Starting with the pioneering work of El-Abiad and Nagappan[7] and that of Gless[8], there have been a number of researchers who have developed or proposed various Liapunov functions for studying transient stability of power systems, although many of the Liapunov functions either reduce to or are simplified versions of the one developed by El-Abiad and Nagappan[7]. Various techniques have been used by different authors to arrive at the Liapunov functions. Trial and error and energy consideration have been used in references 7, 8, 9, 12 and 25. References 10 and 13 use variable gradient method[29]. Dharma Rao[18, 13] has used Cartwright method, Aizerman method, method of integration by parts[30] and method of Infante and Clark[31]. Zubov's method[32] has been used in references 11 and 26. Recent interest has been to use Popov's frequency criterion[33] as a necessary and sufficient condition for the existence of a special type of Liapunov function of a quadratic form

plus an integral of the nonlinearities[34,35]. Based on this criterion, Kalman's construction procedure[34] has been successfully used to construct Liapunov functions for single machine and multimachine systems by Pai et.al[20-22]. In a parallel development, the results of Moore and Anderson[35] for the stability of multinonlinear systems have been used to get Liapunov functions by Willems[14-17] and Pai and Murthy[23,24] for multimachine systems. The methods that yield better Liapunov functions for single machine system suffer from the limitations that it is very difficult to extend these methods to multimachine systems or systems with more detailed representation of synchronous machines. As this thesis will demonstrate, when more detailed machine models are used, it becomes necessary to modify some of the known methods, to suit the particular problem.

1.5 SOME GENERALITIES

Consider a nth order dynamic process which can be represented in an n-dimensional state space as:

$$\dot{\underline{x}} = A\,\underline{x} + \underline{b}\,f(\sigma)$$

$$\sigma = \underline{c}^T\,\underline{x} \qquad (1.1)$$

where A is a nxn real constant matrix, $\underline{b}$ and $\underline{c}$ real constant n-vectors and $\underline{x}$ is the n-state vector. σ is a scalar. $f(\sigma)$, the nonlinearity which is also called 'characteristic'[36], is an arbitrary, single-valued

piecewise continuous real function, defined for all real values of σ and satisfying

$$f(\Theta) = 0$$

$$0 \le \frac{f(\sigma)}{\sigma} \le k \tag{1.2}$$

where k is either a finite positive number or infinity. In the latter case it is equivalent to the inequality $\sigma f(\sigma) \geq 0$. It will be further assumed that the system (1.1) satisfies the usual conditions necessary to ensure the existence and uniqueness of a solution for all $t \geq 0$, starting from any initial state $\underline{x}(0) = \underline{x}^o$.

If in (1.1), the matrix A is strictly Hurwitz, by which is meant that the roots of the characteristic equation

$$|sI - A| = 0 \tag{1.3}$$

all lie on to the left of the imaginary axis of the complex s-plane, the system is known as 'the principal case'[36] or sometimes called 'direct control' system[37]. Suppose the characteristic equation (1.3) has one zero root. It is then possible by means of a linear nonsingular transformation to transform the original system (1.1) into a system

$$\begin{aligned} \dot{\underline{y}} &= F\,\underline{y} - \underline{g}\,f(\sigma) \\ \dot{\xi} &= -\,f(\sigma) \\ \sigma &= \underline{h}^T\,\underline{y} + \rho\,\xi \end{aligned} \tag{1.4}$$

where now F is a (n-1)x(n-1) real constant matrix which is Hurwitz, i.e.

$$|sI - F| = 0 \qquad (1.5)$$

has all its roots on the left half of s-plane. $\underline{g}$ and $\underline{h}$ are real (n-1)-vectors, $\underline{y}$ is a (n-1)-state vector and ξ the nth state variable. When system (1.1) is such that the characteristic equation (1.3) has one zero root and the remaining (n-1) roots on the left half of s-plane, the system is known as the 'simplest particular case'[36] or 'indirect control'[37]. In general, if the system (1.1) is such that some of the characteristic roots of (1.3) are on the imaginary axis (including zero roots) and the rest in the left half of the s-plane, it is called a particular case of (1.1) or briefly 'particular case'. For particular cases, the class of admissible nonlinear functions have to be narrowed down to the inequality

$$0 < \frac{f(\sigma)}{\sigma} \le k \qquad (\sigma \neq 0) \quad \text{if } k \text{ is finite,}$$

and $$\sigma f(\sigma) > 0 \qquad (\sigma \neq 0) \quad \text{if } k \text{ is infinite} \qquad (1.6)$$

The systems (1.1) and (1.4) are equivalent as long as the transformation is linear and nonsingular and therefore, the stability properties of (1.4) are identical to those of (1.1)[36,37].

1.6 POWER SYSTEM PROBLEM

A single machine infinite bus system under transient condition can first be formulated in the form of (1.1) for phase B referred in section 1.2. It will be observed, that the power system problem belongs to the particular case of (1.1). By suitable transformation the problem will be represented in the form of (1.4). In general, a power system formulation may contain more than one nonlinearity either because of more detailed representation of synchronous machine or because of the presence of more number of machines. The most general form of the system equations would then be

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{f}(\underline{\sigma})$$

$$\underline{\sigma} = C^T\,\underline{x} \tag{1.7}$$

where $\underline{\sigma}$ and $\underline{f}(\underline{\sigma})$ are now m-vectors, and B and C are nxm matrices. Furthermore, if the power system has k synchronous machines (called hereafter a k-machine system), one would discover, that the matrix A in (1.7) is such that the characteristic equation (1.3) will now have k zero roots and the rest of the roots on the left half of s-plane. It is still possible to transform this most general system into what may now be called the matrix version of (1.4) as

$$\dot{\underline{y}} = F\,\underline{y} - G\,\underline{f}(\underline{\sigma})$$

$$\dot{\underline{\xi}} = -N\,\underline{f}(\underline{\sigma})$$

$$\underline{\sigma} = H^T\,\underline{y} + P\,\underline{\xi} \tag{1.8}$$

where F is now a (n-k)x(n-k) real constant strictly Hurwitz matrix, G, N, H and P are real constant (n-k)xm, kxm, (n-k)xm and mxk matrices respectively. $\underline{y}$ and $\underline{\xi}$ are (n-k) and k-vectors respectively and $\underline{f}$ and $\underline{\sigma}$ being m-vectors. For convenience we will call (1.4) and (1.8) as the 'standard forms'.

The differential equations for the phase B are cast in the form of (1.4) or (1.8) as the case may be. We now seek a Liapunov function consisting of a 'quadratic form plus an integral of the nonlinearities' type. The relevant Liapunov theorem on which we base our stability analysis of power system is given in Appendix A. It is appropriate to note that the nonlinearity $f(\sigma)$ in a power system problem, does not satisfy the 'sector condition' (1.6) in the whole state space. However, around the post-fault equilibrium point which will be taken as the origin of the state space, there is a region in which the condition (1.6) is satisfied and therefore the stability properties can be studied by the Liapunov method in this region. Indeed, this region of stability will be of interest since the critical clearing time is computed with the aid of this.

1.7 SCOPE OF THE THESIS

A brief account of the work that is reported in the following pages is outlined below.

The synchronous machine model including transient saliency is dealt with in the second chapter. A single machine connected to an infinite bus is considered first. A Liapunov function is developed using Kalman's construction procedure[34]. This Liapunov function is used to determine the stability region and the critical clearing time. A detailed parameter analysis is carried out for a numerical example. For the sake of comparison a similar parameter analysis is also made with transient saliency neglected. A two-machine system with both machines modelled with transient saliency considered, is studied next. When dealing with a multimachine system, two distinct cases arise: one in which the damping is 'uniform', by which is meant the damping ratio D_i/M_i is the same for all machines; and the other case wherein the damping is'nonuniform', meaning thereby that the damping ratios for different machines are different. The two-machine system referred to above, is studied for both cases. A general k-machine system is then briefly discussed, indicating the difficulties involved in systematically arriving at a Liapunov function for this system.

A single machine system with greater details in machine representation, is the theme of the third chapter. First a model with flux-decay effect and voltage reulator

action included, is investigated. This problem turns out to be multinonlinear system. Further the nonlinearities happen to be of the 'multi-argument' type. Therefore, a direct application of a matrix version of Kalman's construction procedure[38,39] is not possible. However, a modification of this procedure with the incorporation of results from Desoer and Wu[40] on stability of nonlinear systems, provides a systematic method to arrive at a Liapunov function. With the Liapunov function developed, the effects of voltage regulator parameters and flux-decay on the critical clearing time are illustrated through a numerical example. Next a more complete model of a synchronous machine to include transient saliency, flux-decay, voltage regulator and governor dynamics, is used to derive a generalized Liapunov function. This Liapunov function naturally subsumes the previously derived Liapunov functions as special cases.

Chapter four is concerned with the multimachine system with flux-decay effect included. The resulting model is not amenable to the procedures discussed in the previous chapters. A version of 'integration by parts' method is found suitable. Using this technique, Liapunov functions are derived for two-machine and three-machine systems. These results are extended by induction to a general k-machine system.

Chapter five deals with a power system with transfer conductance. A two-machine system is formulated and a Liapunov function derived through the matrix version of Kalman's construction procedure. A three-machine system is then formulated.

A brief review of the results, identification of some unsolved problems and suggestions for future work form the sixth and the concluding chapter.

CHAPTER II

EFFECT OF SALIENCY ON TRANSIENT STABILITY

2.1 INTRODUCTION

In most studies of transient stability, a simple model of the synchronous machine represented by constant voltage back of a transient reactance is used. This reactance is assumed constant, regardless of the angular position of the armature reaction with respect to the rotor. This is equivalent to assuming that the direct axis and quadrature axis transient reactances are equal ($x'_q = x'_d$) and that a slowly decaying component of rotor flux linkages exists in the direct as well as the quadrature axis. In reality however, most of the rotor circuit flux linkages decay relatively rapidly except those supported by the main field winding which has a relatively large time constant. It is therefore more correct that only the main field winding flux linkages are held constant during the transient period and the other more rapidly decaying flux linkages in the rotor are neglected. The synchronous machine may then be said to have 'transient saliency' ($x'_q \neq x'_d$)[2]. It is to be noted that transient saliency is present in both salient pole and cylindrical rotor machines, although the latter has

practically no synchronous saliency. The synchronous saliency is reflected in the electrical output in the form of an additional term for the salient pole machine. We will consider salient pole machines in the subsequent sections.

The effect of saliency has been discussed in great detail in earlier works, notably in references 2 and 41. Many others have used salient-pole model in their study of transient stability[11,42,26,27]. The aim of the present chapter is to develop a systematic procedure for constructing a Liapunov function for a system model including transient saliency. Liapunov functions are developed for a single machine-infinite bus system and a two-machine system using Kalman's construction procedure[34]. For the latter both uniform and nonuniform damping are considered. These Liapunov functions can be used to find regions of stability[43]. The critical fault clearing time for each system is computed by numerical integration of the faulted system equations until the trajectory reaches the boundary of the respective stability region. A detailed parameter analysis is carried out for the single machine case. The complexity of the problem when extended to a k-machine system with transient saliency is indicated.

2.2 SINGLE MACHINE CONNECTED TO INFINITE BUS

2.2.1 Formulation of the Problem:

The dynamic equation of the single machine system can be written as

$$M \ddot{\delta} + D \dot{\delta} = P_I - P_e \tag{2.1}$$

where P_I, the mechanical input, is assumed to remain constant. The damping coefficient is also assumed to be constant. Variable damping can be taken into account as discussed in reference 41 at the expense of more analytical computations[26,27]. The electrical power output P_e of a salient pole machine connected to an infinite bus can be computed from[44,2]

$$P_e = \frac{E_Q^2}{Z_{11}} \sin \alpha_{11} + \frac{E_Q e_t}{Z_{12}} \sin(\delta - \alpha_{12}) \tag{2.2}$$

where E_Q is the voltage behind the quadrature axis synchronous reactance and e_t is the infinite bus voltage. Z_{11} and Z_{12} are the driving point and transfer impedance respectively, with quadrature axis synchronous reactance; $\alpha_{11} = \frac{\pi}{2} - \theta_{11}$; $\alpha_{12} = \frac{\pi}{2} - \theta_{12}$; θ_{11} and θ_{12} are the impedance angles of Z_{11} and Z_{12} respectively (radians). The voltage E_Q is related to a voltage E'_q which is the quadrature axis component of the voltage behind the direct axis transient reactance (E'_q is a voltage considered proportional to field flux linkages) by the relation[2]

$$E'_q = [1 + \frac{(x'_d - x_q)}{Z_{11}} \sin \theta_{11}] E_Q - \frac{(x'_d - x_q)}{Z_{12}} e_t \sin(\delta + \theta_{12}) \tag{2.3}$$

If resistances are neglected, the electrical power in terms of E'_q and e_t can be shown to be[2] (Refer to the phasor diagram of Figure 2.1).

$$P_e = \frac{E'_q \ e_t}{x'_d + x_e} \sin \delta + \frac{e_t^2 (x'_d - x_q)}{2(x_q + x_e)(x'_d + x_e)} \sin 2\delta$$

$$= P_1 \sin \delta + P_2 \sin 2\delta$$

$$\text{where } P_1 = \frac{E'_q \ e_t}{(x'_d + x_e)} \ ; \quad P_2 = \frac{e_t^2 (x'_d - x_q)}{2(x_q + x_e)(x'_d + x_e)} \tag{2.4}$$

Here x_e is the external reactance through which the machine is connected to the infinite bus. It should be noted that P_e in (2.1) has to be evaluated with P_1 and P_2 computed for the particular condition of operation such as prefault, post-fault or faulted condition, with the corresponding network impedances. The equilibrium points of the post-fault system where $\ddot{\delta} = \dot{\delta} = 0$, can be determined by solving for δ from

$$P_I - P_1 \sin \delta - P_2 \sin 2\delta = 0 \tag{2.5}$$

Let δ^s be the value of δ at the post-fault stable equilibrium point.

Defining

$$x = \delta - \delta^s. \qquad (2.6)$$

the equation (2.1) can be rewritten as

$$M\ddot{x} + D\dot{x} = P_I - P_1 \sin(x+\delta^s) - P_2 \sin(2x+2\delta^s) \qquad (2.7)$$

Now define the state variables x_1 and x_2 as

$$\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \end{aligned} \qquad (2.8)$$

Let $\sigma = x_1$ and

$$f(\sigma) = \frac{P_1}{M}\sin(\sigma+\delta^s) + \frac{P_2}{M}\sin 2(\sigma+\delta^s) - \frac{P_I}{M} \qquad (2.9)$$

Using (2.8) and (2.9), the system equations in the state space can be put as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -D/M \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} f(\sigma)$$

$$\sigma = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (2.10)$$

This system can be represented by the block diagram shown in Figure 2.2. The nonlinear function defined by (2.9) satisfies the sector condition $f(0) = 0$ and $\sigma f(\sigma) > 0$ for $\sigma \neq 0$ for the range (see Figure 2.3a and 2.3b)

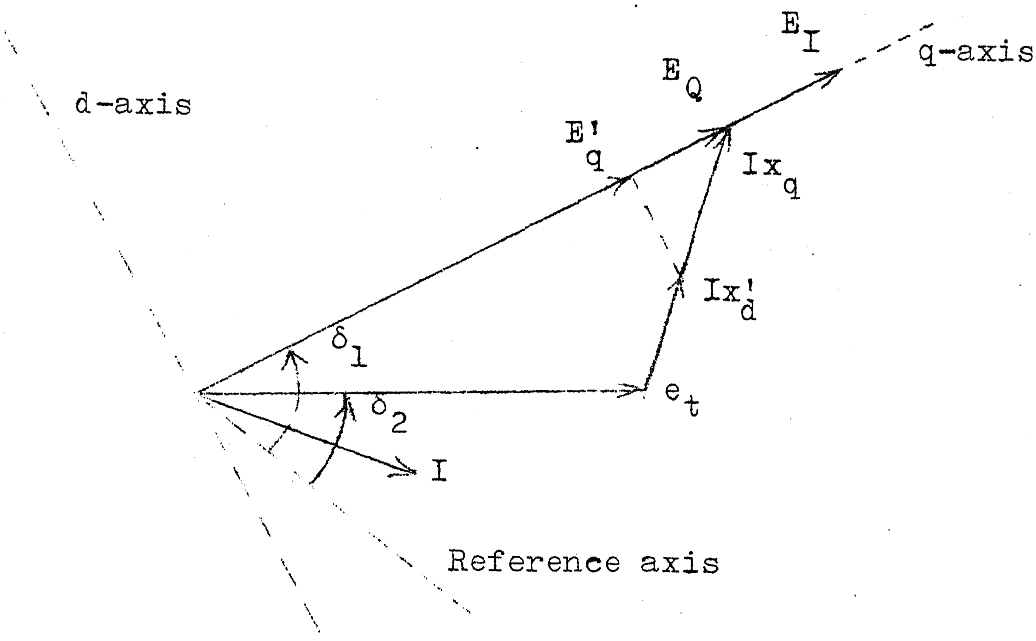

FIG. 2.1 PHASOR DIAGRAM OF A SALIENT POLE GENERATOR.

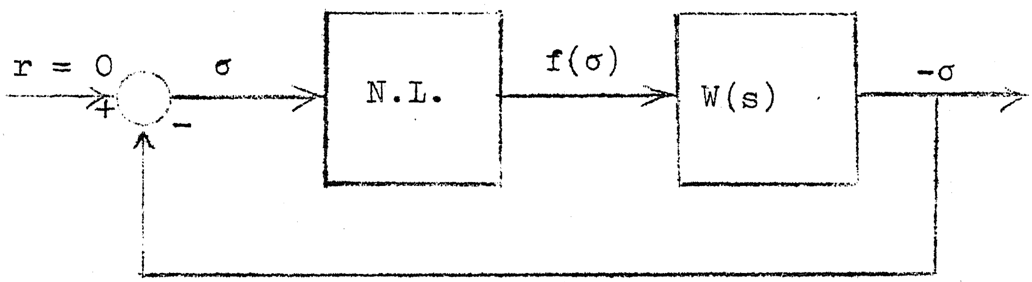

FIG. 2.2 SYSTEM WITH A SINGLE NONLINEARITY.

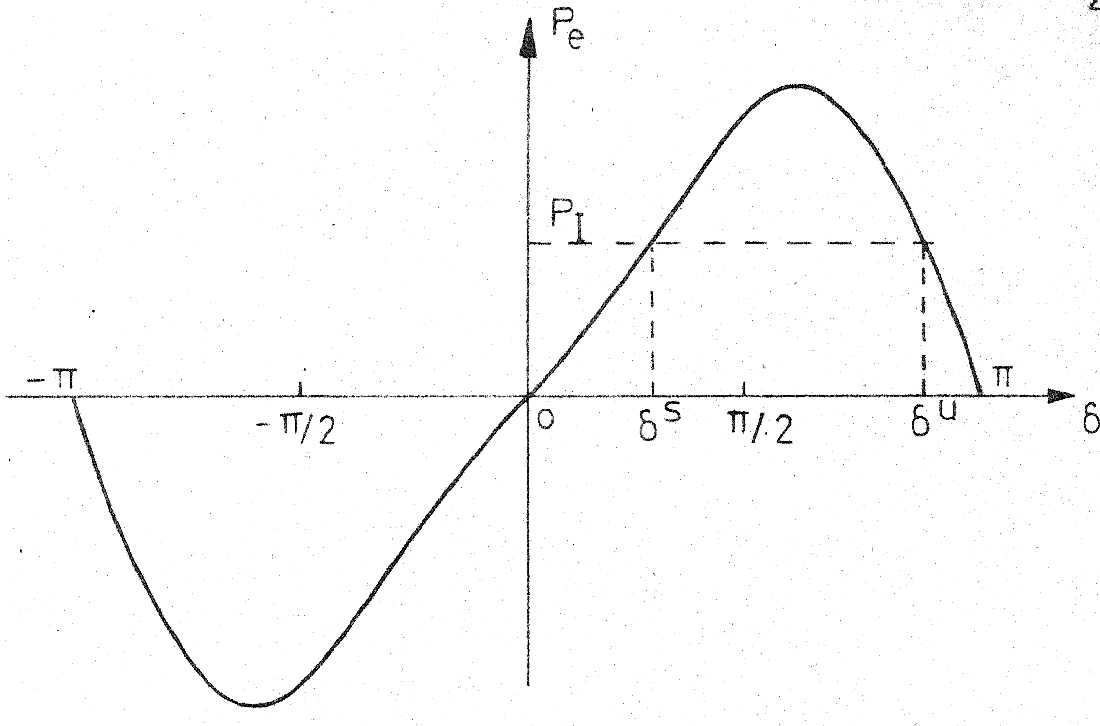

FIG. 2·3 a POWER-ANGLE CURVE OF A SALIENT POLE SYNCHRONOUS MACHINE

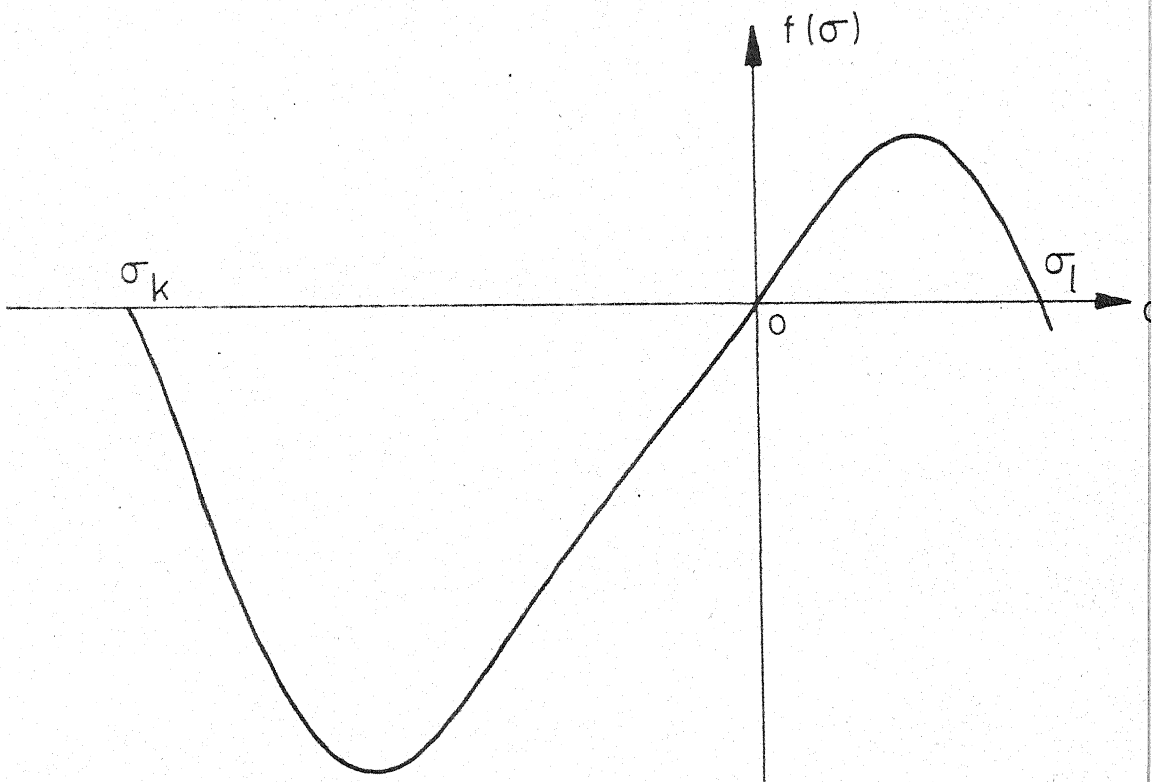

FIG. 2·3 b NONLINEARITY $f(\sigma)$

$$-(2\pi-\delta^{u}-\delta^{s}) < \sigma < (\delta^{u}-\delta^{s}) \qquad (2.11)$$

Popov's criterion[33] can therefore be applied for analysing the system in the region where the sector condition is satisfied by the nonlinearity. Indeed the operating region of our interest falls within this region. System (2.10) is similar in form to (1.1). We transform this into the form (1.4) by defining a new state variable[22]

$$\xi = \frac{D}{M} x_1 + x_2 \qquad (2.12)$$

With the introduction of this state variable and eliminating x_1 from (2.10) , the system equations reduce to

$$\dot{x}_2 = -\frac{D}{M} x_2 - f(\sigma)$$

$$\dot{\xi} = -f(\sigma)$$

$$\sigma = -\frac{M}{D} x_2 + \frac{M}{D}\xi \qquad (2.13)$$

A Liapunov function is now to be derived for the system (2.13) which is in the 'indirect control' form of the Lur'e system[38].

2.2.2 Liapunov Function for the Lur'e Problem with a Single Nonlinearity:

Consider the Lur'e problem defined by[38]

$$\dot{\underline{x}} = F\,\underline{x} - \underline{g}\,f(\sigma)$$

$$\dot{\xi} = -f(\sigma) \qquad (2.14)$$

$$\sigma = \underline{h}^T\,\underline{x} + \rho\,\xi$$

where F is a real nxn strictly Hurwitz matrix, $\underline{x}$, $\underline{g}$ and $\underline{h}$ are real n-vectors and σ, ξ and ρ are real scalars. $f(\sigma)$ is a real valued continuous function which satisfies a sector condition

$$f(0) = 0 \quad \text{and} \quad \sigma f(\sigma) > 0 \qquad \sigma \neq 0 \qquad (2.15)$$

For the system (2.14) Popov[33] enunciated the following theorem:

'Assume F is stable and that $\rho > 0$. Then (2.14) is globally asymptotically stable if the condition

$$\text{Re}\{(2\alpha\rho + j\omega\beta)[\underline{h}^T(j\omega I-F)^{-1}\underline{g} + \frac{\rho}{j\omega}]\} \geq 0$$

$$\text{for all real } \omega \qquad (2.16)$$

holds for $2\alpha\rho = 1$ and some $\beta \geq 0$'. Differentiating the third equation in (2.14), a system equivalent to (2.14) can be written as

$$\dot{\underline{x}} = F\underline{x} - \underline{g}\, f(\sigma)$$
$$\dot{\sigma} = \underline{h}^T F \underline{x} - (\underline{h}^T \underline{g} + \rho)\, f(\sigma) \qquad (2.14^*)$$

Popov showed that for the system (2.14*) (or equivalently for (2.14)) the most general form of a Liapunov function which is of the kind described as 'quadratic form plus the integral of the nonlinearity', takes the form

$$V(\underline{x}, \sigma) = \underline{x}^T L \underline{x} + \alpha(\sigma - \underline{h}^T \underline{x})^2 + \beta \int_0^\sigma f(\sigma)\, d\sigma \qquad (2.17)$$

He further established that the frequency condition (2.16) is necessary for the existence of a Liapunov function of the type given in (2.17). Kalman subsequently showed[34] that (2.16) is also sufficient for the existence of the above type of V-function. Furthermore, utilizing and extending the results of Popov[33], Yakubovitch[45] and LaSalle[46], Kalman developed a systematic procedure for constructing the above type of Liapunov function. The procedure which relies on what has come to be known as Kalman-Yakubovitch Lemma can be summarized as follows[47]:

Define $\quad Z(s) = (2\alpha\ \rho + \beta s)\ W(s) \qquad (2.18)$

where W(s), which is the transfer function of the linear part of the system relating σ and $f(\sigma)$ (see Figure 2.2) is given by

$$W(s) = \underline{h}^T\ (sI - F)^{-1}\ \underline{g} + \frac{1}{s}\ \rho \qquad (2.19)$$

The frequency condition (2.16) is now equivalent to the following:

$$\tfrac{1}{2}[Z(j\omega) + Z(-j\omega)] \geq 0 \quad \text{for all real } \omega \qquad (2.20)$$

If now for the system (2.14), the above frequency condition is satisfied, then a Liapunov function exists. This Liapunov function can be constructed in the following steps:

(i) Factorize the sum $\tfrac{1}{2}[Z(s) + Z(-s)]$ in the form

$$\tfrac{1}{2}[Z(s) + Z(-s)] = T(-s)\ T(s) \qquad (2.21)$$

(ii) Determine the scalar γ from

$$\gamma = \beta(\rho + \underline{h}^T \underline{g}) \tag{2.22}$$

(iii) Solve for the elements of n-vector $\underline{u}$ from the identity

$$T(s) - \sqrt{\gamma} \equiv -\underline{u}^T(sI - F)^{-1} \underline{g} \tag{2.23}$$

(iv) Now solve the Liapunov matrix equation

$$F^T L + L F = - \underline{u}\, \underline{u}^T \tag{2.24}$$

for the symmetric positive semidefinite (possibly positive definite) nxn matrix L. Knowing L, the Liapunov function is now obtained as in (2.17). The Liapunov function thus obtained will satisfy the sign definite properties ($V > 0$ and $\dot{V} < 0$) required for the absolute stability provided the nonlinearity satisfies the sector condition (2.15). Kalman imposes the requirement that (F, $\underline{g}$) be completely controllable and (F, $\underline{h}^T$) be completely observable. These restrictions have been relaxed subsequently by Meyer[48].

2.2.3 Application to the Problem in Section 2.2.1:

The power system problem described by (2.13) is seen to be identical to the Lur'e system (2.14), with the following identities:

$$F = -\frac{D}{M} \; ; \quad g = 1 \; ; \; h^T = -\frac{M}{D} \; ; \; \rho = \frac{M}{D} \tag{2.25}$$

With these identities, the transfer function W(s) given by (2.19) for the linear part is seen to be

$$W(s) = \frac{1}{s(s+\lambda)} \tag{2.26}$$

where $\lambda = \frac{D}{M}$

Choosing

$$\alpha = D/2M \tag{2.27}$$

the left hand side of the frequency condition (2.20) becomes

$$\tfrac{1}{2}[Z(j\omega) + Z(-j\omega)] = \frac{(\lambda\beta - 1)}{\omega^2 + \lambda^2} \tag{2.28}$$

Thus the frequency condition (2.20) will be satisfied if $\lambda\beta \geq 1$ (2.29)

β can be so chosen as to satisfy this condition.

Now the Liapunov function V is constructed:

(i) With Z(s) as in (2.19), it can be verified that

$$\tfrac{1}{2}[Z(s) + Z(-s)] = \frac{\lambda\beta - 1}{(-s^2 + \lambda^2)}$$

Factorizing as in (2.21), we have

$$T(s) = \sqrt{(\lambda\beta - 1)}/(s+\lambda) \tag{2.30}$$

(ii) Using (2.22) it can be seen

$$\gamma = 0 \tag{2.31}$$

(iii) Noting that in this problem u is a scalar, we have on solving (2.23)

$$u = -\sqrt{(\lambda\beta - 1)} \tag{2.32}$$

(iv) Again L is a scalar now (say ℓ). The Liapunov matrix equation (2.24) now becomes a scalar equation and solving this we have

$$\ell = \frac{(\lambda\beta - 1)}{2\lambda} \tag{2.33}$$

Hence the Liapunov function as given by (2.17) is

$$V(x_2,\sigma) = \frac{(\lambda\beta-1)}{2\lambda}x_2^2 + \frac{\lambda}{2}(\sigma + \frac{M x_2}{D})^2 + \beta \int_0^{\sigma} f(\sigma)d\sigma \tag{2.34}$$

This is the most general form of Liapunov function with $\beta \geq 1/\lambda$. For the particular choice of $\beta = 1/\lambda$ we have

$$V(x_2,\sigma) = \frac{D}{2M}(\sigma + \frac{M x_2}{D})^2 + \frac{M}{D}\int_0^{\sigma} f(\sigma)d\sigma \tag{2.35}$$

With $f(\sigma)$ as in (2.9), (2.35) becomes

$$V(x_2,\sigma) = \frac{D}{2M}(\sigma + \frac{M x_2}{D})^2 + \frac{M}{D}[\frac{P_1}{M}(\cos\delta^s - \cos(\sigma+\delta^s)) +$$

$$\frac{P_2}{2M}(\cos 2\delta^s - \cos 2(\sigma+\delta^s)) - \frac{P_I}{M}\sigma] \tag{2.36}$$

For the power system model under consideration, the nonlinearity violates the sector condition as it leaves the first and the third quadrants at values of σ equal

to σ_ℓ and σ_k respectively (Figure 2.3b). For such a system there is only a finite region of asymptotic stability around the origin. This region R around the origin is defined by the inequality

$$V < C \tag{2.37}$$

where $\quad C = \min (C_1, C_2)$

with $\quad C_1 = \min\limits_{\sigma=\sigma_k} V(x_2, \sigma)$

$$C_2 = \min_{\sigma=\sigma_\ell} V(x_2, \sigma) \tag{2.38}$$

Determination of C_1 and C_2 using Lagrange multipliers[43] has been discussed by Pai et.al.[20]. For the system under consideration, $C_2 < C_1$ and it can be shown[47] that

$$C_2 = \frac{M}{D} \int_0^{\sigma_\ell} f(\sigma)d\sigma \tag{2.39}$$

where $\sigma_\ell = \delta^u - \delta^s$. δ^u is the value of δ at the post-fault unstable equilibrium point. Using (2.9) and (2.39), we have

$$C_2 = \frac{P_1}{D}[\cos\delta^s - \cos\delta^u] + \frac{P_2}{2D}(\cos 2\delta^s - \cos 2\delta^u) - \frac{P_I}{D}(\delta^u - \delta^s) \tag{2.40}$$

The region of asymptotic stability is given by

$$\frac{M}{2D}\left(\frac{D}{M}\sigma + x_2\right)^2 + \frac{1}{D}\Big[P_1(\cos\delta^s - \cos(\sigma+\delta^s)) +$$

$$\frac{P_2}{2}(\cos 2\delta^s - \cos 2(\sigma+\delta^s)) - P_I\,\sigma\Big] < C_2 \tag{2.41}$$

For a more general choice of β satisfying $\beta > 1/\lambda$, corresponding Liapunov function can be obtained.

Critical Clearing Time:

The prefault operating point δ_0 is first computed for the initial operating condition from load flow. The stable and unstable post-fault equilibrium points are determined[7] by the well-established methods such as steepest descent method, Fletcher Powell or Fletcher Reeves method. Then, starting from this prefault equilibrium state, the system differential equation (2.7) is numerically integrated for the faulted condition till the boundary of the inequality (2.41) is reached. This gives the critical clearing time.

Parameter Variation:

Equation (2.7) corresponds to a particular set of parameters. While the system parameters M and D are decided upon when designing, P_1 and P_2 are dependent on the initial excitation and location of the fault while P_I depends on the load demand at the instant of fault. In an overall system planning and study, it may be of interest to investigate how variations in these parameters reflect on the critical clearing time. Such a parameter analysis is illustrated through a numerical example in the next section. The influence of other parameters such as the various reactances, which also affect stability, can also be similarly investigated.

Numerical Example:

Figure 2.4 shows a power system with a three phase fault on one of the two transmission lines[1]. Each transmission line has a reactance of 0.2 p.u. The 25 MVA generator has the following constants:

$H = 2.76$ MJ/MVA ; $x_d = 1.0$; $x_q = 0.65$; $x_d' = 0.3$;

Frequency = 60 Hz.

The transient stability of the above system is investigated for various situations. On the lines indicated in the previous section, a parameter analysis[49] is made. For an assumed three phase to ground fault at the middle of one of the transmission lines, the values of D, M and P_I are varied. Similarly keeping a set of fixed values for D, M and P_I, the fault location is varied. For each set of parameters $V(x_2, \sigma)$ and C_2 are computed. The effects of all these variations on the critical clearing time t_c are investigated as shown in Figures 2.5 to 2.8. In studying the effect of fault location, a parameter α is introduced for convenience. α is a number between 0 and 1 and is defined as the ratio of the amplitude of the fundamental component of the electrical power during the faulted state to that during the prefault condition. Thus α in some way, is a measure of the reduction in the power output due to the fault which in turn, is dependent on the

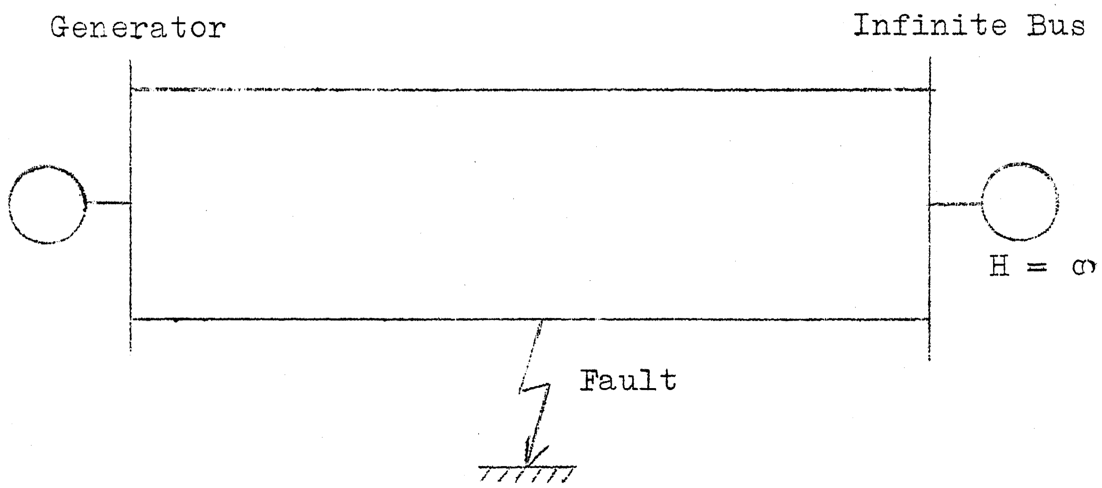

FIG. 2.4 A SYNCHRONOUS MACHINE - INFINITE BUS SYSTEM.

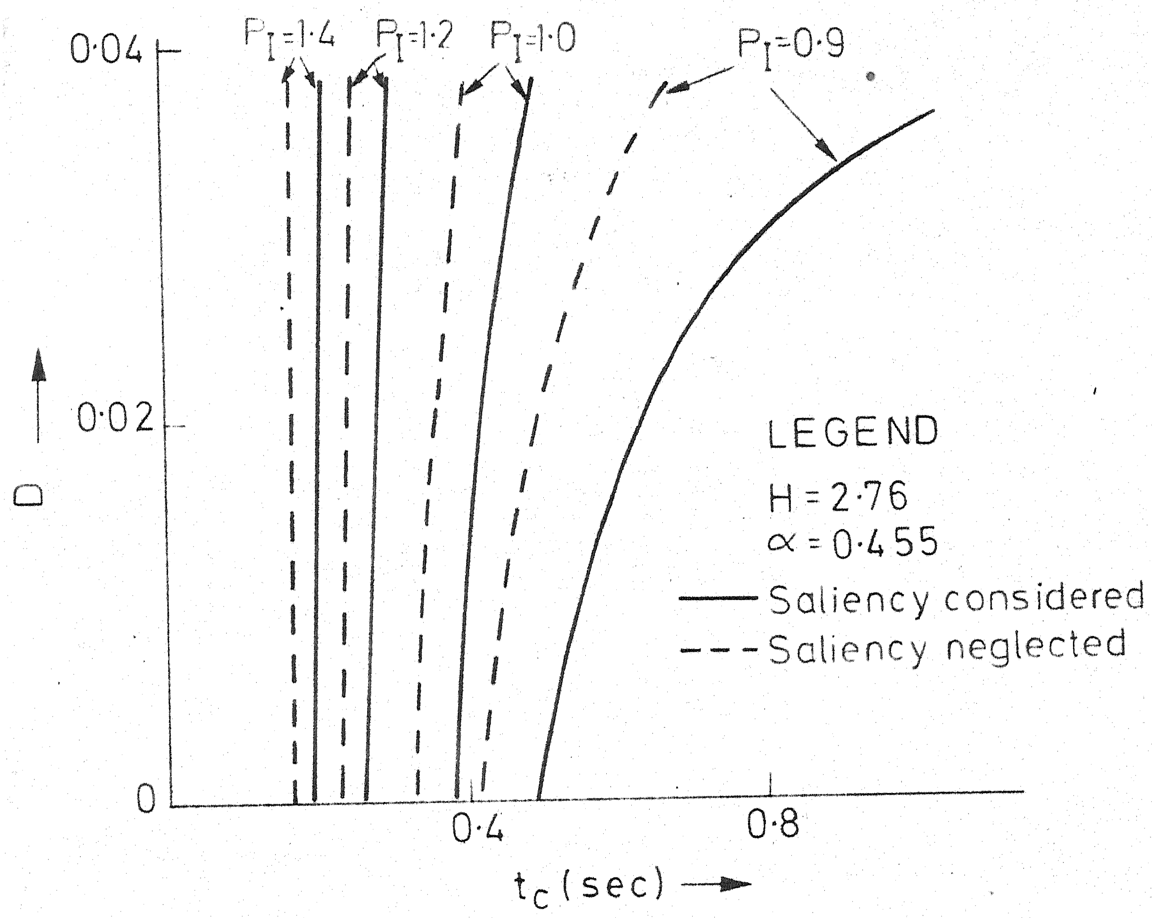

FIG. 2·5 EFFECT OF DAMPING COEFFICIENT (D) ON CRITICAL CLEARING TIME (t_c)

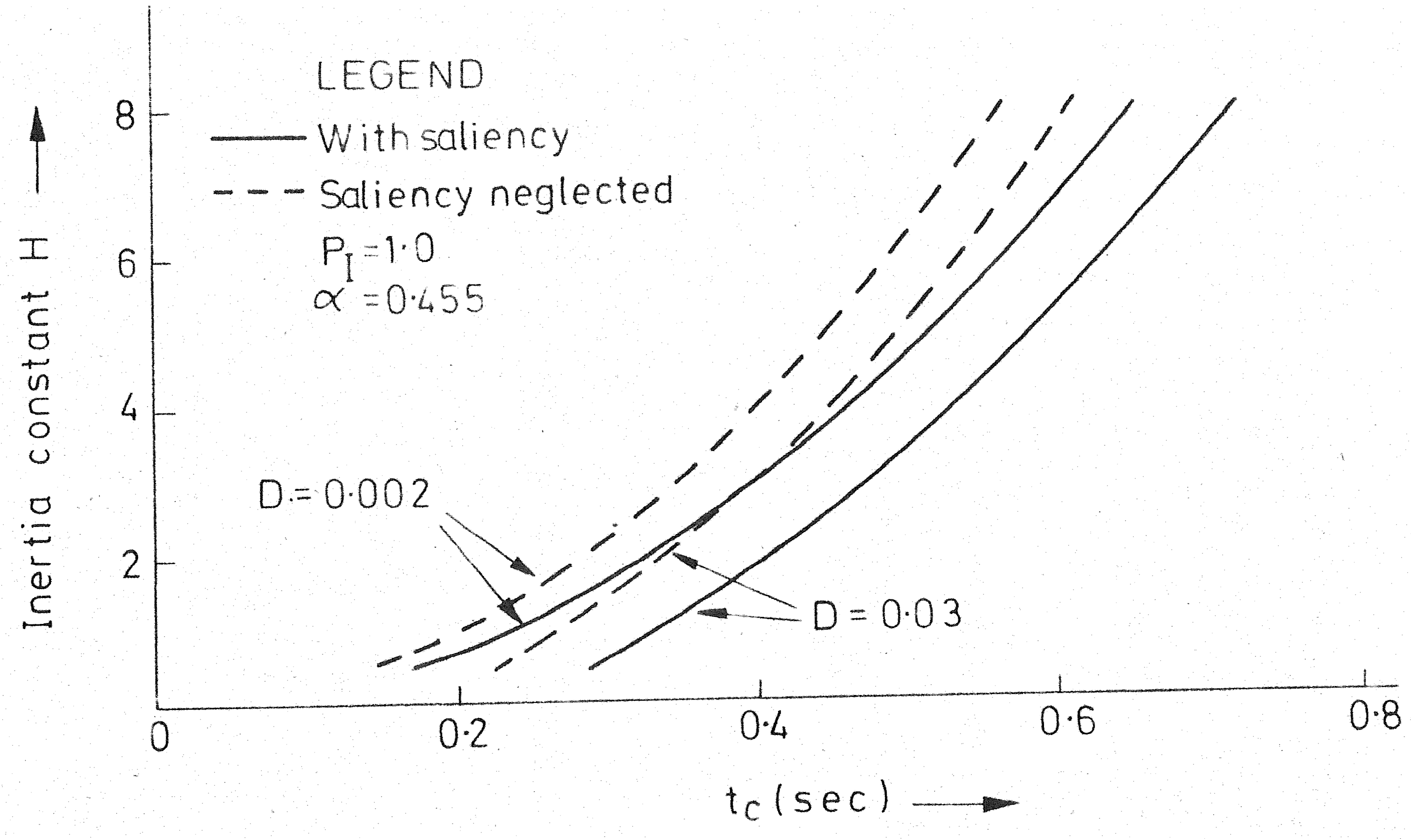

FIG. 2.6 EFFECT OF INERTIA ON t_c

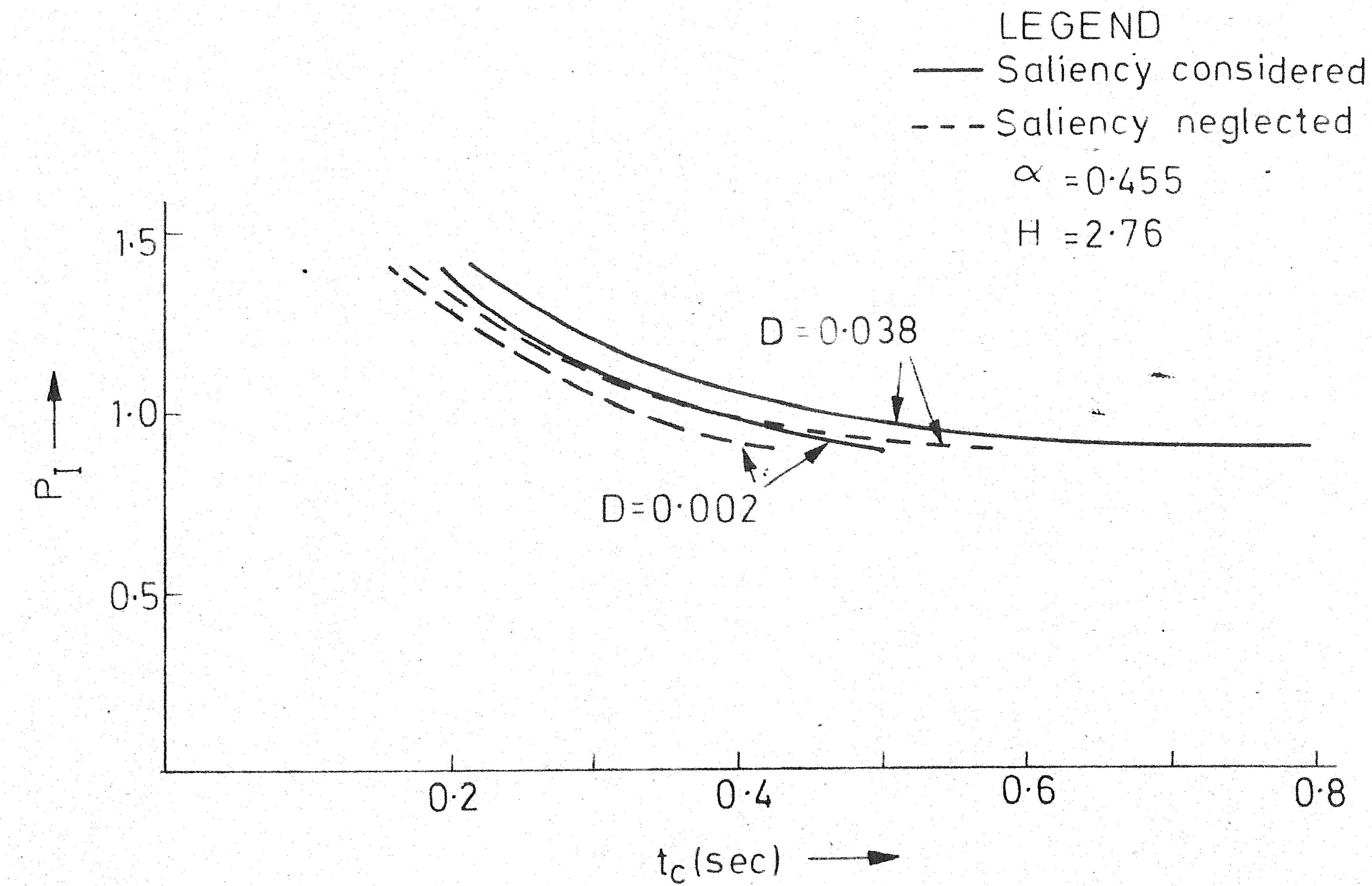

FIG. 2·7 EFFECT OF P_I ON t_c

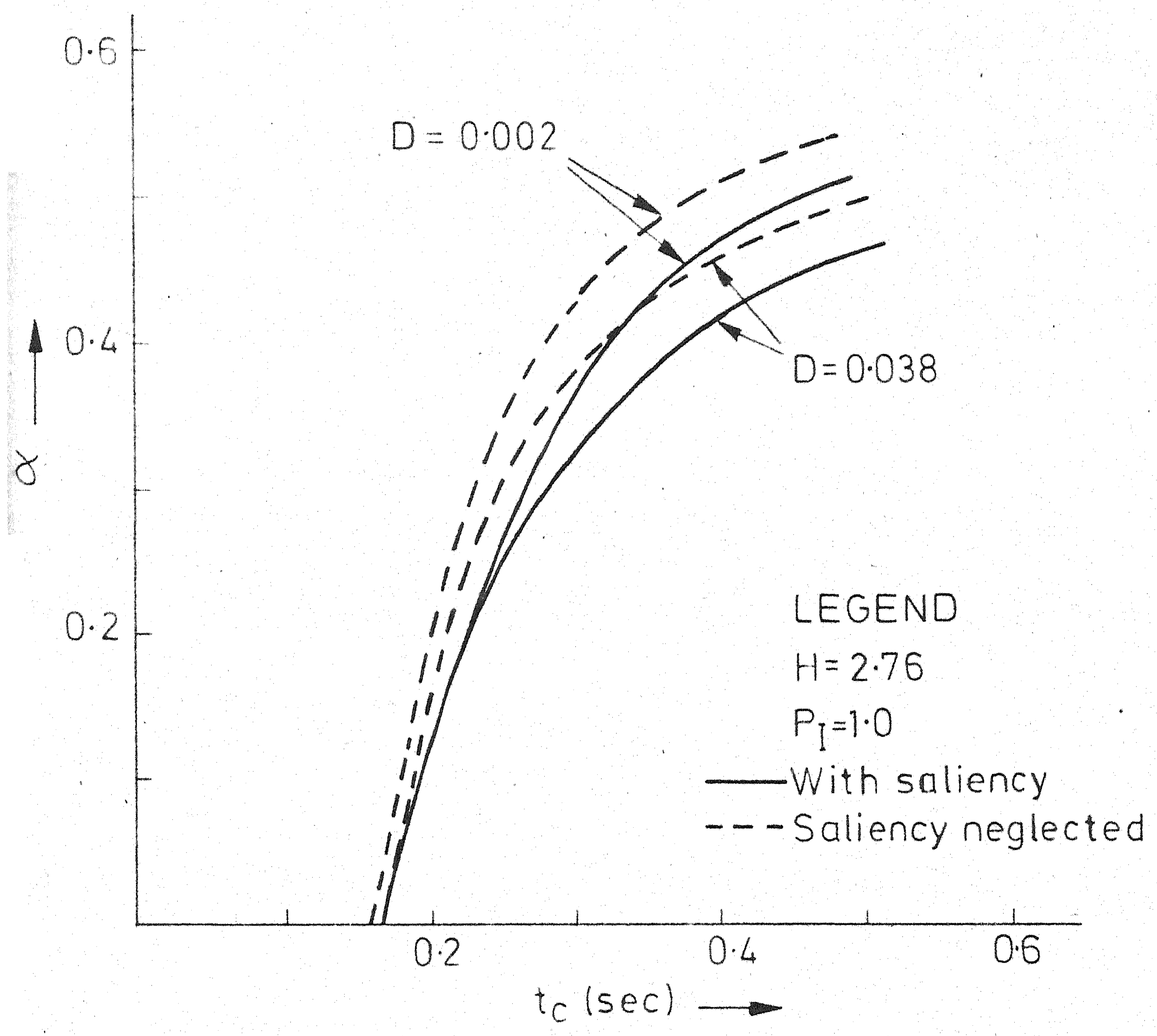

FIG. 2·8 EFFECT OF α ON t_c

location of the fault. For comparison, the critical clearing time for the above conditions, with transient saliency neglected, are also computed and plotted.

Discussion on the Results:

The parameter analysis conducted as described above, yields very useful information. Figures 2.5 to 2.8 show clearly, the effect of the variation of the various parameters on the critical clearing time t_c. Saliency in general has an effect of increasing critical clearing time. The extent of increase however, depends on the other parameters also. The effect of damping for instance is also to increase t_c. But this effect is more pronounced as the input power P_I is reduced or equivalently if the initial load on the machine is less. The results indicate that if the machine operates with outputs less than 1 p.u., it may be worthwhile to consider introducing higher damping so as to increase the critical clearing time. However, if the machine is required to operate at 1 p.u. or higher output power, an increase in damping coefficient D does not seem to significantly affect t_c (Figure 2.5). These observations may be difficult to translate in terms of physical parameters since damping is known to be not constant in general. Nevertheless it offers some insight into the otherwise highly complicated phenomena. Figure 2.6 shows how t_c varies with inertia

constant H, which is linearly related to M, appearing in the differential equation. A higher value of inertia constant results in a higher value of t_c. This is consistent with physical considerations; for, a machine with higher inertia constant will take more time to reach the critical clearing angle, implying thereby a greater t_c. Figure 2.7 shows effect of P_I on t_c with H, D and α held constant. As can be expected, lower the value of P_I, the greater is the value of t_c. Here it is observed that the effect of saliency is more pronounced at lower value of P_I. Thus, at lower loads on the generator, saliency seems to increase t_c considerably. The effect of varying the location of fault - which corresponds to varying α - with H, D and P_I held constant, is shown in Figure 2.8. It is observed that higher value of α (i.e. less reduction in electric power output during the faulted condition or which is equivalent to a fault farther away from the buses), greater is the value of t_c. This is understandable, because, the farther the fault away from the buses, less severe it is. It can also be seen from the graphs, that saliency has greater influence on t_c at higher values of α. In practice P_I is dictated by the prefault load demand and α signifies the fault location. These parameters cannot be preassigned. However, one can prepare a chart for ready reference predicting the behaviour of the system for various contingencies. On

the other hand, inertia constant and damping and to some degree the relevant reactances are controllable parameters. It may therefore be worthwhile to make a study like the above during planning and designing a system. Some of the above conclusions have been reported in Crary[2] through extensive simulation. However, Liapunov method in conjunction with digital computer affords a quick way of obtaining extensive results on effects of parameter variation.

2.3 TWO-MACHINE SYSTEM

2.3.1 Formulation:

The theory discussed in the foregoing sections can be extended to the two-machine system, with both machines having saliency. The equations of motion for the two machines can be written as

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i = P_{mi} - P_{ei} \qquad (i=1,2) \tag{2.42}$$

where the subscript i stands for the ith machine. Using these subscripts for voltages, currents and reactances of the corresponding machines, it can be shown[2] that the power output of machines 1 and 2 will be given respectively by

$$P_{e1} = \frac{E_{Q1}^2}{Z_{11}} \sin \alpha_{11} + \frac{E_{Q1}\, E_{Q2}}{Z_{12}} \sin(\delta_1 - \delta_2 - \alpha_{12})$$

$$P_{e2} = \frac{E_{Q2}^2}{Z_{22}} \sin \alpha_{22} + \frac{E_{Q1}\, E_{Q2}}{Z_{12}} \sin(\delta_2 - \delta_1 - \alpha_{12}) \qquad (2.43)$$

E_{Q1} and E_{Q2} are the voltages back of the quadrature axis synchronous reactances of machines 1 and 2 respectively. Z_{ii} $(i=1,2)$ and Z_{12} are the driving point and transfer impedance respectively. It is to be noted here that $Z_{ij} = Z_{ji}$ and also $\alpha_{ij} = \alpha_{ji}$. δ_1 and δ_2 are the angles of the voltages E_{Q1} and E_{Q2} respectively from the synchronously rotating reference axis. The voltages E_{Q1} and E_{Q2} are to be determined from the following two equations:

$$E'_{q1} = a_{11}\, E_{Q1} + [a_{12} \sin(\delta_{12} + \theta_{12})] E_{Q2}$$

$$E'_{q2} = a_{21}\, E_{Q1} \sin(\delta_{21} + \theta_{12}) + a_{22}\, E_{Q2} \qquad (2.44)$$

where E'_{q1} and E'_{q2}, the voltages proportional to field flux linkages of the machines 1 and 2 respectively, are known and held constant (by the constant flux linkage theorem). Further $\delta_{ij} = \delta_i - \delta_j$. The coefficients are given by

$$a_{11} = 1 + \frac{(x'_{d1} - x_{q1})}{Z_{11}} \sin \theta_{11} \; ; \quad a_{12} = \frac{(x_{q1} - x'_{d1})}{Z_{12}}$$

$$a_{21} = \frac{(x_{q2} - x'_{d2})}{Z_{12}} \; ; \quad a_{22} = 1 + \frac{(x'_{d2} - x_{q2})}{Z_{22}} \sin \theta_{22} \qquad (2.45)$$

Neglecting the resistances for the sake of simplicity, we have $\theta_{11} = \theta_{22} = \theta_{12} = \pi/2$ and $\alpha_{11} = \alpha_{22} = \alpha_{12} = 0$. Then equation (2.43) reduces to

$$P_{e1} = \frac{E_{Q1}\, E_{Q2}}{Z_{12}} \sin \delta_{12} = - P_{e2} \tag{2.46}$$

and (2.44) reduces to

$$\begin{aligned} E'_{q1} &= a_{11}\, E_{Q1} + a_{12}\, E_{Q2} \cos \delta_{12} \\ E'_{q2} &= a_{21}\, E_{Q1} \cos \delta_{21} + a_{22}\, E_{Q2} \end{aligned} \tag{2.47}$$

with a_{12} and a_{21} as given in (2.45), and

$$a_{11} = 1 + (x'_{d1} - x_{q1})/Z_{11} \;; \quad a_{22} = 1+(x'_{d2}-x_{q2})/Z_{22} \tag{2.48}$$

Z_{ii} and Z_{ij} are now the driving point and transfer reactances respectively. Let δ_1^s and δ_2^s be the post-fault stable values of δ_1 and δ_2 respectively.

Define

$$\begin{aligned} x_1 &= \delta_1 - \delta_1^s \\ x_2 &= \delta_2 - \delta_2^s \\ x_3 &= \dot{x}_1 = \dot{\delta}_1 \\ x_4 &= \dot{x}_2 = \dot{\delta}_2 \end{aligned} \tag{2.49}$$

Further, let $\sigma = x_1 - x_2$ (2.50)

Define now the nonlinearity

$$f(\sigma) = \frac{E_{Q1}\ E_{Q2}}{Z_{12}} \sin(\sigma + \delta_{12}^{s}) - P_{m1} \tag{2.51}$$

where $\delta_{12}^{s} = \delta_{1}^{s} - \delta_{2}^{s}$. It is to be noted that E_{Q1} and E_{Q2} in (2.51) are obtained from (2.47) which shows that these voltages are functions of δ_{12} and hence σ. The expression for $f(\sigma)$ is thus seen to be much more complex than for the single-machine system. However, it can be verified that $f(\sigma)$ does satisfy the sector conditions around the origin, so that Popov's criterion can be applied in a region around the origin. With equations (2.49) and (2.51), the system dynamics (2.42) can be cast into the form:

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= [-D_1 x_3 - f(\sigma)]/M_1 \\
\dot{x}_4 &= [-D_2 x_4 + f(\sigma)]/M_2
\end{aligned} \tag{2.52}$$

with σ as in (2.50). The system (2.52) can be transformed into the standard form (1.8) by the change of variables

$$\begin{aligned}
\xi_1 &= D_1\ x_1 + M_1\ x_3 \\
\xi_2 &= D_2\ x_2 + M_2\ x_4
\end{aligned} \tag{2.53}$$

Replacing x_1 and x_2 with these new variables, the system can be represented in the form

$$\begin{aligned} \dot{\underline{x}} &= F\,\underline{x} - \underline{g}\,f(\sigma) \\ \dot{\underline{\xi}} &= -\underline{d}\,f(\sigma) \\ \sigma &= \underline{h}^T\,\underline{x} + \underline{\rho}^T\,\underline{\xi} \end{aligned} \tag{2.54}$$

where

$$\underline{x} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \quad ; \quad \underline{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \tag{2.55}$$

$$\text{and} \quad F = \begin{bmatrix} -D_1/M_1 & 0 \\ 0 & -D_2/M_2 \end{bmatrix} \quad ; \quad \underline{g} = \begin{bmatrix} 1/M_1 \\ -1/M_2 \end{bmatrix} \quad ;$$

$$\underline{d} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} ; \quad \underline{h} = \begin{bmatrix} -M_1/D_1 \\ M_2/D_2 \end{bmatrix} \quad ; \quad \rho = \begin{bmatrix} 1/D_1 \\ -1/D_2 \end{bmatrix} \tag{2.56}$$

The transfer function of the linear part of the above system is given by

$$W(s) = \underline{h}^T(sI - F)^{-1}\,\underline{g} + \underline{\rho}^T\,\underline{d}/s \tag{2.57}$$

Define now

$$Z(s) = (2\alpha\,\underline{\rho}^T\,\underline{d} + \beta\,s)\,W(s) \tag{2.58}$$

Note that unlike in Section 2.2.1, here $\underline{\rho}$ is a vector. $\underline{\rho}^T\,\underline{d}$ is a positive scalar though, and we can conveniently choose α such that $2\alpha\,\underline{\rho}^T\,\underline{d} = 1$. As in Section 2.2.2,

a frequency criterion given by (2.20) can be applied here also for the existence of a Liapunov function of the form (2.17). The construction procedure then follows the same way as in that section.

2.3.2 Liapunov function:

As discussed in Chapter I, in a multimachine system two distinct cases, 'nonuniform' and 'uniform' damping, arise. Both these cases will be dealt with for the two-machine system[50].

Case (i) Nonuniform Damping:

$$\text{Let } D_1/M_1 = \lambda_1 \text{ and } D_2/M_2 = \lambda_2 \text{ and } \lambda_1 \neq \lambda_2 \tag{2.59}$$

The transfer function derived from (2.57) using the appropriate matrices and vectors from (2.56), is seen on simplification, to be

$$W(s) = \frac{(1/M_1 + 1/M_2)s + (\lambda_1/M_2 + \lambda_2/M_1)}{s(s+\lambda_1)\ (s + \lambda_2)} \tag{2.60}$$

Define

$$1/M_1 + 1/M_2 = a \ ; \ \lambda_1/M_2 + \lambda_2/M_1 = b \tag{2.61}$$

Choose

$$\alpha = \frac{D_1 D_2}{2(D_1+D_2)} \tag{2.61}$$

so that $2\alpha\ \underline{\rho}^T\ \underline{d} = 1.$ With this, we have

$$Z(s) = \frac{b+(a+\beta b)s + a\beta s^2}{s(s+\lambda_1)(s+\lambda_2)} \tag{2.62}$$

Using (2.62), we get

$$\tfrac{1}{2}[Z(j\omega) + Z(-j\omega)] = [\lambda_1\lambda_2(a+b\beta)-b(\lambda_1+\lambda_2)+\omega^2(a\beta(\lambda_1+\lambda_2) - (a+b\beta))]/[(\omega^2+\lambda_1^2)(\omega^2+\lambda_2^2)] \qquad (2.63)$$

Therefore, to satisfy the Popov's frequency condition (2.20) or equivalently for (2.63) to be nonnegative, it is necessary that

$$\lambda_1\lambda_2(a+b\beta) - b(\lambda_1 + \lambda_2) \geq 0 \qquad (2.64)$$

and

$$a\beta(\lambda_1 + \lambda_2) - (a + b\beta) \geq 0 \qquad (2.65)$$

Substituting for a and b from (2.61) the condition (2.64) can be seen to reduce to

$$\frac{\lambda_1\lambda_2}{M_1 M_2}(D_1+D_2)[\beta + \frac{M_1 + M_2}{D_1 + D_2} - (1/\lambda_1 + 1/\lambda_2)] \geq 0 \qquad (2.66)$$

This can always be satisfied by appropriately choosing β. Clearly, different values of β will result in different V-functions. A possible choice is

$$\beta = 1/\lambda_1 + 1/\lambda_2 \qquad (2.67)$$

With this choice of β, the condition (2.65) will simplify to $(M_1\lambda_2^2 + M_2\lambda_1^2)/(\lambda_1\lambda_2 M_1M_2) \geq 0$ which is always true. Indeed the left hand side of (2.65) is strictly positive.

Construction of V-function:

Steps involved in constructing V-function, are the same as in 2.2.2 except that, in the steps (i) and (ii), $\underline{\rho}^T \underline{d}$ is to be used instead of mere ρ.

(i) With Z(s) as given in (2.62) it can be shown that

$$\tfrac{1}{2}[Z(s)+Z(-s)] = \frac{k_1 - k_2 s^2}{(-s^2+\lambda_1^2)(-s^2+\lambda_2^2)} \tag{2.68}$$

where

$$k_1 = D_1 D_2 (M_1+M_2)/(M_1 M_2)^2; \quad k_2 = (M_1\lambda_2^2+M_2\lambda_1^2)/D_1 D_2 \tag{2.69}$$

Factorizing (2.68) in the form given in (2.21), we have

$$T(s) = (\sqrt{k_1} + \sqrt{k_2}\, s)/[(s+\lambda_1)(s+\lambda_2)] \tag{2.70}$$

(ii) Similar to (2.22), now we have

$$\gamma = \beta(\underline{\rho}^T \underline{d} + \underline{h}^T \underline{g})$$

which on substitution of appropriate vectors results in

$$\gamma = 0 \tag{2.71}$$

(iii) Using (2.23), $\underline{u}$ is to be determined. Let $\underline{u}$ be of the form

$$\underline{u} = \begin{bmatrix} u_{11} \\ u_{21} \end{bmatrix} \tag{2.72}$$

Then equation (2.23)upon substitution yields

$$T(s) \equiv \frac{\left(\frac{u_{21}\lambda_1}{M_2} - u_{11}\frac{\lambda_2}{M_1}\right) + \left(\frac{u_{21}}{M_2} - \frac{u_{11}}{M_1}\right)s}{(s+\lambda_1)(s+\lambda_2)} \tag{2.73}$$

With T(s) as given in (2.70), we obtain from (2.73) by equating coefficients of like terms

$$-u_{11}\lambda_2/M_1 + u_{21}\lambda_1/M_2 = \sqrt{k_1}$$

$$-u_{11}/M_1 + u_{21}/M_2 = \sqrt{k_2} \tag{2.74}$$

Solving simultaneously the two equations in (2.74), we have

$$u_{11} = M_1(\sqrt{k_1} - \lambda_1\sqrt{k_2})/(\lambda_1 - \lambda_2)$$

$$u_{21} = M_2(\sqrt{k_1} - \lambda_2\sqrt{k_2})/(\lambda_1 - \lambda_2) \tag{2.75}$$

(iv) Let the matrix L in the Liapunov matrix equation (2.24) be of the form

$$L = \begin{bmatrix} \ell_{11} & \ell_{12} \\ \ell_{12} & \ell_{22} \end{bmatrix} \tag{2.76}$$

Substitution of F, u and L in (2.24) results in the identities:

$$-2\,\ell_{11}\lambda_1 = -u_{11}^2$$

$$-2\,\ell_{22}\lambda_2 = -u_{21}^2$$

$$-\ell_{12}(\lambda_1+\lambda_2) = -u_{11}u_{21} \tag{2.77}$$

Substitution of u_{11} and u_{21} from (2.75) in (2.77), will give on simplification:

$$\ell_{11} = \frac{M_1^2(k_1 + \lambda_1^2 k_2 - 2\lambda_1 \sqrt{(k_1 k_2)})}{2\lambda_1(\lambda_1-\lambda_2)^2}$$

$$\ell_{22} = \frac{M_2^2(k_1 + \lambda_2^2 k_2 - 2\lambda_2 \sqrt{(k_1 k_2)})}{2\lambda_2(\lambda_1 - \lambda_2)^2}$$

$$\ell_{12} = \frac{M_1 M_2}{(\lambda_1+\lambda_2)(\lambda_1-\lambda_2)^2}[k_1+\lambda_1\lambda_2 k_2-(\lambda_1+\lambda_2)\sqrt{(k_1 k_2)}] \tag{2.78}$$

L can be verified to be positive definite. The Liapunov function is now given by

$$V(\underline{x}, \sigma) = (\ell_{11} + \frac{\alpha}{\lambda_1^2})x_3^2 + (\ell_{22} + \frac{\alpha}{\lambda_2^2})x_4^2 + (2\ell_{12} - \frac{2\alpha}{\lambda_1\lambda_2})x_3 x_4 +$$

$$\alpha\sigma^2 + \frac{2\alpha}{\lambda_1}\sigma x_3 - \frac{2\alpha}{\lambda_2}\sigma x_4 + \beta \int_0^\sigma f(\sigma)d\sigma \tag{2.79}$$

where α, β and $f(\sigma)$ are as defined in (2.61), (2.67) and (2.51) respectively.

Case (ii) Uniform Damping:

Consider the case when

$$D_1/M_1 = D_2/M_2 = \lambda \tag{2.80}$$

In this case it is possible to combine the two equations in (2.42) into a single differential equation of the following form:

$$\ddot{\delta}_{12} + \lambda \dot{\delta}_{12} = \frac{1}{M_o} (P_{m1} - P_{e1}) \tag{2.81}$$

where

$$M_o = M_1 M_2/(M_1+M_2) \tag{2.82}$$

Other symbols have the same meanings as in the previous case.

Define the state variables

$$x_1 = \delta_{12} - \delta_{12}^s$$
$$x_2 = \dot{x}_1 \tag{2.83}$$

Define also

$$f(\sigma) = \frac{1}{M_o} [\frac{E_{Q1} E_{Q2}}{Z_{12}} \sin(\sigma + \delta_{12}^s) - P_{m1}] \tag{2.84}$$

where

$$\sigma = x_1 \tag{2.85}$$

The voltages E_{Q1} and E_{Q2} are to be determined as before from (2.47). With the above variables, the system equations will now be:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = - \lambda x_2 - f(\sigma) \tag{2.86}$$

with $\sigma = x_1$. Equations (2.86) are exactly similar to the system equations (2.10) for the single machine - infinite bus system except for the difference in definitions of the state variables and nonlinearity.

Hence with $\alpha = \lambda/2$ and $\beta = 1/\lambda$, we have a Liapunov function

$$V(x_2,\sigma) = \frac{\lambda}{2}(\sigma + x_2/\lambda)^2 + \frac{1}{\lambda}\int_0^{\sigma} f(\sigma)d\sigma \qquad (2.87)$$

with x_2 and σ as defined in (2.83) and (2.85) respectively. In the case when machine 2 corresponds to an infinite bus, (2.87) and (2.35) are identical. With the Liapunov function available, the stability study of the system can be carried out including the determination of the critical clearing time.

2.4 EXTENSION TO k-MACHINE SYSTEM

The method discussed in the previous sections can be extended in principle to a k-machine system, including transient saliency. The form of the set of differential equations describing the system will be

$$M_i\ddot{\delta}_i + D_i\dot{\delta}_i = P_{mi} - P_{ei} \quad (i=1,2,\ldots,k) \qquad (2.88)$$

P_{ei} is the electrical power output of the ith machine and in this case, the determination of this quantity involves considerable labour. It can be shown[44,2] that the electrical power output of the ith machine is given by

$$P_{ei} = \frac{E_{Qi}^2}{Z_{ii}}\sin\alpha_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{k} \frac{E_{Qi}E_{Qj}}{Z_{ij}}\sin(\delta_{ij} - \alpha_{ij}) \qquad (2.89)$$

where E_{Qi} and E_{Qj} are the voltages back of the quadrature

axis synchronous reactance of the ith and jth machines respectively. Z_{ii} and Z_{ij} are, as before, the driving point and transfer impedances respectively. α_{ii} and α_{ij} have the same meaning as before. It is to be noted that $Z_{ij} = Z_{ji}$ and $\delta_{ij} = \delta_i - \delta_j$. The voltages $E_{Q1}, E_{Q2}, \ldots, E_{Qk}$ are to be determined from the following set of k simultaneous equations:

$$E'_{qi} = [1 + \frac{(x'_{di} - x_{qi})}{Z_{ii}} \sin \theta_{ii}] E_{Qi} -$$

$$(x'_{di} - x_{qi}) \sum_{\substack{j=1 \\ j \neq i}}^{k} \frac{E_{Qj}}{Z_{ij}} \sin(\delta_{ij} + \theta_{ij}) \quad (i=1,2,\ldots,k) \tag{2.90}$$

If we neglect now the resistances as in the previous cases, (2.89) and (2.90) reduce to

$$P_{ei} = \sum_{\substack{j=1 \\ j \neq i}}^{k} \frac{E_{Qi} E_{Qj}}{Z_{ij}} \sin \delta_{ij} \qquad (i = 1,2,\ldots,k) \tag{2.91}$$

$$E'_{qi} = [1 + \frac{x'_{di} - x_{qi}}{Z_{ii}}] E_{Qi} - (x'_{di} - x_{qi}) \sum_{\substack{j=1 \\ j \neq i}}^{k} \frac{E_{Qj}}{Z_{ij}} \cos \delta_{ij}$$

$$(i=1,2,\ldots,k) \tag{2.92}$$

Thus E_{Qi} $(i = 1,2,\ldots,k)$ can be expressed in terms of the known voltages E'_{qi} $(i = 1,2,\ldots,k)$ which are proportional to the field flux linkages and substituting for these voltages, the power P_{ei} $(i = 1,2,\ldots,k)$ can be

determined for solving the differential equations (2.88). It is to be noted however, that (2.92) is a set of k algebraic equations in E_{Qi} $(i = 1,2,\ldots,k)$ whose coefficients are functions of δ_{ij}. Furthermore, when the order k is high, an explicit expression for E_{Qi} in terms of the known voltages E'_{qi} $(i = 1,2,\ldots,k)$ is highly complicated and when substituted into the power expression (2.91), results in a nonlinearity that is very difficult - if not impossible - to handle. Therefore, at the present time, no attempt is made to develop a Liapunov function for multimachine system with saliency effect included.

2.5 CONCLUSION

In this chapter, the saliency effect has been included in the stability analysis of power systems, in some depth. A single machine infinite bus has been discussed in great detail. A Liapunov function V has been derived and this V-function has been utilized for a parameter analysis of a typical system. This example illustrates clearly, the power of the Liapunov approach in making large number of studies with very little effort and time compared to the conventional step-by-step method. A two-machine system is then formulated. Both the 'nonuniform' and 'uniform' damping cases have been dealt with and appropriate V-functions

derived. Studies similar to those conducted for the single machine-infinite bus system can be easily done for this system also. The difficulty of extending to a k-machine system, where k is large, is pointed out.

The models for which V-functions have been successfully developed, can be allowed to include further details such as flux decay effect, voltage regulator and governor dynamics. We will see the case of a single machine-infinite bus system with such details in the next chapter.

CHAPTER III

SINGLE-MACHINE SYSTEM WITH IMPROVED MACHINE MODEL

3.1 INTRODUCTION

In this chapter a system with a single machine connected to an infinite bus, referred to hereafter as a 'single machine system' for brevity, is investigated further. As mentioned at the end of the last chapter, the synchronous machine is modelled to include further details such as flux decay, voltage regulator and governor dynamics. At first a model incorporating only flux decay and voltage regulator is studied. It is observed that the system now has multiple nonlinearities. Furthermore, the nonlinearities belong to the class of 'multiargument' type because of which a direct application of the matrix version of Kalman's construction procedure[38,39] is not possible. However, a modification of this construction procedure by incorporating the results of Desoer and Wu[40] provides us with a method to obtain a V-function. Using this V-function, a parameter analysis is carried out to investigate the effect of the main parameters of the voltage regulator. The same construction procedure is then extended to a system model wherein saliency effect and governor dynamics are also included in addition to the flux decay and voltage

regulator action. The Liapunov function derived for this model is seen to be most general and naturally the Liapunov functions derived earlier happen to be special cases of this V-function.

3.2 SINGLE MACHINE SYSTEM WITH FLUX DECAY AND VOLTAGE REGULATOR

Several authors[1,2,41,44,10,51-53] have dealt with the effects of flux decay and voltage regulator action on the transient stability of synchronous generator. Indeed these effects are being included in many of the digital simulation studies. It is only appropriate therefore that these effects are included in stability studies via Liapunov method.

The dynamic equation governing the flux decay is simple and straightforward. Inclusion of this in the system equations therefore poses no particular problem. However, the nonlinearities involved in the system dynamics now get changed to the 'multiargument' type, the implication of which has already been mentioned in Section 3.1.

The voltage regulator action on the other hand, can be represented in several ways. The exact model of a voltage regulator renders the problem difficult for solution through Liapunov approach. We will therefore approximate it by a model having an exponential type of

response[54]. The finite time that elapses before the voltage regulator begins to act following a fault, is neglected partly because it is very small and partly for the sake of mathematical simplicity. With this representation of voltage regulator, the system equations become nonautonomous. Siddiqee used this model as it is and his attempts to derive a V-function through known methods of construction seems to have failed[12]. Indeed, it appears that the well-established construction procedures that have been successfully employed to derive V-functions for autonomous systems, cannot be extended to nonautonomous systems. Fortunately however, with the above model for voltage regulator, it is possible to convert the nonautonomous problem of the power system into an autonomous one, by a change of variable. This enables to derive a Liapunov function using a modified version of Kalman's construction procedure.

3.2.1 Formulation of the Problem:

Neglecting saliency and resistances the rotor dynamics of a single machine connected to an infinite bus can be written as

$$M \ddot{\delta} + D \dot{\delta} = P_m - \frac{E'_q E_B}{x_e + x'_d} \sin \delta \tag{3.1}$$

where E_B is now the infinite bus voltage. The flux decay effect can be represented through the variation of E'_q which is given by

$$\dot{E}'_q = [E_{ex} - (E'_q + (x_d - x'_d) i_d)]/T'_o \tag{3.2}$$

where

$$i_d = (E'_q - E_B \cos \delta)/(x_e + x'_d) \tag{3.3}$$

Substituting (3.3) into (3.2) and simplifying,

$$\dot{E}'_q = \frac{E_{ex}}{T'_o} - \frac{E'_q}{T'_o}\left(1 + \frac{x_d - x'_d}{x_e + x'_d}\right) + \frac{(x_d - x'_d)\, E_B}{(x_e + x'_d)\, T'_o} \cos \delta \tag{3.4}$$

The voltage regulator action on the exciter voltage is approximated by[54]

$$E_{ex} = E_c - K_v\, E_o\, e^{-\eta_4 t} \tag{3.5}$$

where

$$E_c = E_o(1 + K_v) \tag{3.6}$$

with E_o as the initial exciter voltage at the time of fault. Define

$$x_1 = \delta - \delta^s$$

$$x_2 = \dot{x}_1 = \dot{\delta}$$

$$x_3 = E'_q - e$$

$$x_4 = E_{ex} - E_c = -K_v\, E_o\, e^{-\eta_4 t} \tag{3.7}$$

where e is the post fault steady state value of E'_q.

Define also

$$\eta_1 = \frac{x_e+x_d}{(x_e + x'_d)T'_o} \quad ; \quad \eta_2 = \frac{(x_d - x'_d)E_B}{(x_e + x'_d)T'_o} \quad ;$$

$$\eta_3 = 1/T'_o \quad ; \quad \eta_4 = 1/T_v \quad ; \quad K_1 = E_B/(x_e+x'_d) \tag{3.8}$$

Using (3.7) and (3.8), equation (3.4) can be rewritten after some manipulations as

$$\dot{x}_3 = -\eta_1 x_3 + \eta_3 x_4 - \eta_2(\cos \delta^s - \cos(x_1 + \delta^s)) \tag{3.9}$$

From (3.7)

$$\dot{x}_4 = -\eta_4 \, x_4 \tag{3.10}$$

Define now

$$f_1(\underline{\sigma}) = \frac{K_1}{M} (\sigma_2 + e) \sin(\sigma_1 + \delta^s) - \frac{P_m}{M}$$

$$f_2(\sigma_1) = \eta_2[\cos \delta^s - \cos(\sigma_1 + \delta^s)] \tag{3.11}$$

where

$$\sigma_1 = x_1 \quad ; \quad \sigma_2 = x_3 \tag{3.12}$$

Making use of the above relationships, the overall system can be represented by a set of first order differential equations as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\lambda \, x_2 - f_1(\underline{\sigma})$$

$$\dot{x}_3 = -\eta_1 x_3 + \eta_3 x_4 - f_2(\sigma_1)$$

$$\dot{x}_4 = -\eta_4 x_4 \tag{3.13}$$

Defining a new state variable ξ as

$$\xi = \lambda\, x_1 + x_2 \tag{3.14}$$

and by a change of variable, the system can be represented by the alternate form

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & -\eta_1 & \eta_3 \\ 0 & 0 & -\eta_4 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_1(\underline{\sigma}) \\ f_2(\sigma_1) \end{bmatrix}$$

$$\dot{\xi} = -[1 \quad 0] \begin{bmatrix} f_1(\underline{\sigma}) \\ f_2(\sigma_1) \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} -M/D & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} M/D \\ 0 \end{bmatrix} \xi \tag{3.15}$$

This is now in the standard form

$$\dot{\underline{y}} = F\, \underline{y} - G\, \underline{f}(\underline{\sigma})$$

$$\dot{\xi} = -\underline{d}^T\, \underline{f}(\underline{\sigma})$$

$$\underline{\sigma} = H^T\, \underline{y} + \underline{\rho}\, \xi \tag{3.16}$$

The nonlinearity $\underline{f}(\underline{\sigma})$ above has one of its components $f_1(\underline{\sigma})$ having two arguments. Furthermore, $f_1(\underline{\sigma})$ violates the sector condition away from the origin. Nevertheless, it is possible to develop a Liapunov function which satisfies the required sign definite properties in a

sufficiently large region around the origin as has been established by the numerical example given in Section 3.4. The transfer function matrix for (3.16) is given by

$$W(s) = H^T(sI - F)^{-1} G + \frac{1}{s} \underline{\rho} \, \underline{d}^T \tag{3.17}$$

Comparing (3.16) and (3.15), and substituting the appropriate matrices and vectors in (3.17), the transfer function matrix for the system (3.15) can be derived to be

$$W(s) = \begin{bmatrix} \frac{1}{s(s+\lambda)} & 0 \\ 0 & \frac{1}{(s+\eta_1)} \end{bmatrix} \tag{3.18}$$

It is to be noted that system (3.15) is a multinonlinear system with two nonlinearities and has a transfer function that is a matrix instead of a scalar. Thus a matrix version of Kalman's construction procedure has to be used. But a direct application of even this procedure is difficult for this case, as the nonlinearities here belong to the class of 'multiargument' type nonlinearities. The difficulty is overcome by suitably adapting Kalman's construction procedure by incorporating some aspects of the work of Desoer and Wu[40].

3.2.2 Liapunov Function for Systems with Multiargument Nonlinearities[55]:

Consider a nth order dynamical system belonging to the simplest particular case characterized by (3.16), where $\underline{y}$ is a (n-1) state vector, ξ is the nth state variable, F is a strictly Hurwitz (n-1)x(n-1) real matrix, G and H are (n-1) x m real matrices, $\underline{d}$ and $\underline{\rho}$ are m-vectors. The m-vector $\underline{f}(\underline{\sigma})$ belongs to a class of multiargument nonlinearities, which implies that each of its components $f_1, f_2, \ldots, f_m$ may be a function of all or some of the m arguments $\sigma_1, \sigma_2, \ldots, \sigma_m$. Now, the following assumptions are made as in Desoer and Wu[40]:

a1: $\underline{f}(\underline{\sigma})$ is continuous and maps R^m into R^m

a2: For some constant real symmetric matrix R and such that $R\,\underline{\rho}\,\underline{d}^T$ is symmetric

$$\underline{\sigma}^T R \underline{\rho}\, \underline{d}^T \underline{f}(\underline{\sigma}) \geq 0 \quad \text{for all } \sigma \in R^m$$

and

$$\underline{f}(\underline{\sigma}) = \underline{0} \quad \text{if} \quad \underline{\sigma} = \underline{0} \tag{3.19}$$

a3: There is a function $V_1 \in C^1$ mapping R^m into R such that $V_1(\underline{\sigma}) \geq 0$ for all $\underline{\sigma} \in R^m$, with

$V_1(\underline{0}) = 0$

and for some constant real matrix Q

$$Q^T \underline{f}(\underline{\sigma}) = \nabla V_1(\underline{\sigma}) \quad \text{for all real } \underline{\sigma} \in R^m \tag{3.20}$$

Now define

$$Z(s) = (2R\underline{\rho}\ \underline{d}^T + Qs)W(s) \tag{3.21}$$

A theorem, which is a modified version of the one due to Anandamohan[47] is given below and proved in the Appendix B.

Theorem:

For the system (3.16), let the assumptions a1-a3 be valid. With constant matrices R and Q defined as in a2 and a3 and Z(s) as in (3.21), the frequency condition

$$\tfrac{1}{2}[Z(j\omega) + Z^T(-j\omega)] \geq 0 \quad \text{for all real } \omega \tag{3.22}$$

is necessary and sufficient for the existence of a Liapunov function

$$V = \underline{y}^T L \underline{y} + (\underline{\sigma} - H^T \underline{y})^T R(\underline{\sigma} - H^T \underline{y}) + V_1(\underline{\sigma}) \tag{3.23}$$

with the derivative along the solutions of the system equations (3.16) being given by

$$\dot{V} = -\| U^T \underline{y} + \Gamma\ \underline{f}(\underline{\sigma}) \|^2 - 2\underline{\sigma}^T R\underline{\rho}\ \underline{d}^T \underline{f}(\underline{\sigma}) \tag{3.24}$$

where L is a (n-1)x(n-1) symmetric positive definite matrix, U is a (n-1)x m matrix and Γ is a m x m matrix.

Procedure for Construction of V-function:

The construction of V-function for the system (3.16) essentially involves choosing the matrices R and Q first, and then arriving at the matrix L through the systematic procedure due to Kalman[34]. Although there is

no precise rule for choosing R and Q, a close examination of the formulation given above gives some general guidelines to arrive at these matrices. For instance, Q and V_1 are related through equation (3.20), so that, once a suitable V_1 is chosen, so as to satisfy assumption a3, Q is fixed. As in the classical Popov-type Liapunov function, V_1 can be chosen to be of the general form of 'an integral of nonlinearities' type. But because of the conditions imposed by a3, it may not always be possible to include all the nonlinearities of the problem, in V_1. It may be possible to have several V_1's and therefore, choice of V_1 is not unique. In such a case one would get correspondingly many Q's and hence many V-functions. It would appear that inclusion of more number of nonlinearities in V_1 would yield better V-functions, implying thereby larger stability regions, as in such V-functions the system would have been reflected to a greater degree of details. Similarly, R can, in general, be chosen arbitrarily, but must be such that it should satisfy assumption a2 and together with the chosen Q, it also should satisfy the frequency condition (3.22). Once R and Q matrices are chosen, the rest of the construction procedure is based on the proof of the matrix version of Kalman-Yacubovich Lemma discussed by Narendra and Goldwyn[38] and Narendra and Neuman[39] and used earlier by Pai and

Anandamohan[22] and Pai[21] for multimachine systems. Essentially, the following steps are involved:

(i) The matrix M(s) is solved for, from the relationship

$$\tfrac{1}{2}[Z(s)+Z^T(-s)] = M^T(-s)M(s) \qquad (3.25)$$

This can be done, in general, by spectral factorization[56-60].

(ii) The matrix Γ is determined from

$$\tfrac{1}{2}Q(H^TG + \underline{\rho}\,\underline{d}^T) + \tfrac{1}{2}(G^TH + \underline{d}\;\underline{\rho}^T)Q^T = \Gamma^T\Gamma \qquad (3.26)$$

(iii) The identity

$$M(s) - \Gamma = -U^T(sI - F)^{-1}\,G \qquad (3.27)$$

is solved for the elements of the (n-1) x m matrix U.

(iv) Finally the Liapunov matrix equation

$$F^TL + L\,F = -U\,U^T \qquad (3.28)$$

is solved for the (n-1) x (n-1) matrix L. With L, R and $V_1(\underline{\sigma})$ known, the Liapunov function can be written as in (3.23), with its derivative given by (3.24).

3.2.3 Application to the Problem of Section 3.2.1:

In the problem discussed in 3.2.1, there are two nonlinearities $f_1(\underline{\sigma})$ and $f_2(\sigma_1)$. Ideally one would wish to include both these nonlinearities in $V_1(\underline{\sigma})$. But

it is observed that this leads to difficulties. The only convenient choice in this case turns out to be

$$V_1 = q \int_0^{\sigma_1} f_1(\underline{\sigma})\, d\,\sigma_1 \tag{3.29}$$

with $q > 0$. Now applying the 'gradient condition' (3.20), the matrix Q is derived as

$$Q = \begin{bmatrix} q & 0 \\ 0 & \frac{qK_1}{M\eta_2} \end{bmatrix} \tag{3.30}$$

R can be in general of the form

$$R = \begin{bmatrix} r_{11} & r_{12} \\ r_{12} & r_{22} \end{bmatrix} \tag{3.31}$$

Comparing (3.15) with (3.16), the matrices F, G, H^T and the vectors $\underline{\rho}$, $\underline{d}$ can be identified for the system under study. It is observed that with the $\underline{\rho}$ and $\underline{d}$ of the system, to make $R\,\underline{\rho}\,\underline{d}^T$ a symmetric matrix, R should be necessarily a diagonal matrix. Thus with the off-diagonal elements of R assumed zeros, the condition (3.19) requires that $r_{11}\ \sigma_1\ f_1(\underline{\sigma}) \geq 0$, i.e.

$$r_{11}\sigma_1[K_1(\sigma_2+e)\ \sin(\sigma_1 + \delta^s) - P_m]/M \geq 0 \tag{3.32}$$

For the problem under consideration, the above condition may get violated in some region around the origin (for example, for a fixed positive value of σ_2), which

in turn would adversely affect the sign definite property of $\dot{V}$. To avoid such a situation, R is chosen as zero matrix:

$$R = 0 \tag{3.33}$$

R and Q have thus been decided upon. With Z(s) as defined in (3.21), it can be shown that

$$\tfrac{1}{2}[Z(j\omega) + Z^T(-j\omega)] = \begin{bmatrix} \dfrac{q\lambda}{\omega^2+\lambda^2} & 0 \\ 0 & \dfrac{(qK_1/M\eta_2)\omega^2}{\omega^2+\eta_1^2} \end{bmatrix} \geq 0$$

(3.34)

λ, K_1 and η_2 all being positive. Now using (3.25), we get on factorization

$$M(s) = \begin{bmatrix} \sqrt{(q\lambda)}/(s+\lambda) & 0 \\ 0 & s\sqrt{(qK_1/M\eta_2)}/(s+\eta_1) \end{bmatrix} \tag{3.35}$$

Next solving (3.26), the matrix Γ is obtained as

$$\Gamma = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{(qK_1/M\eta_2)} \end{bmatrix} \tag{3.36}$$

Let the 3x2 matrix U be of the form

$$U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \end{bmatrix} \qquad (3.37)$$

Substituting the appropriate matrices derived above into (3.27) and solving for U, we have

$$u_{11} = -\sqrt{(q\,\lambda)} \quad ; \qquad u_{12} = u_{21} = 0 \quad ;$$

$$u_{22} = \eta_1 \sqrt{(qK_1/M\eta_2)} \qquad (3.38)$$

The remaining elements u_{31} and u_{32} can be arbitrarily chosen. Choose

$$u_{31} = 0 \quad ; \quad u_{32} = -\frac{\eta_3}{2} \sqrt{(qK_1/M\eta_2)} \qquad (3.39)$$

(Note: This choice of u_{31} and u_{32} has been made deliberately, so as to obtain a diagonal L matrix, for simplicity. In general however, L need not be diagonal).

With the elements of U given as in (3.38) and (3.39), the Liapunov matrix equation (3.28) is now solved for and the matrix L is found to be

$$L = \begin{bmatrix} q/2 & 0 & 0 \\ 0 & qK_1\eta_1/(2M\eta_2) & 0 \\ 0 & 0 & qK_1\eta_3^2/(8M\eta_2\eta_4) \end{bmatrix} \qquad (3.40)$$

which is clearly positive definite.

Now $V_1(\underline{\sigma})$ is calculated from (3.29) after substitution for $f_1(\underline{\sigma})$ from (3.11) together with $P_m = K_1 \, e \sin \delta^s$, and is found to be

$$V_1(\underline{\sigma}) = q\left[\frac{K_1}{M}(\sigma_2+e)(\cos \delta^s - \cos(\sigma_1+\delta^s)) - \frac{K_1}{M} \, e \, \sigma_1 \sin \delta^s\right] \tag{3.41}$$

Without loss of generality, q can be chosen as unity. The Liapunov function will now be given by

$$V(\underline{x}, \underline{\sigma}) = \tfrac{1}{2} x_2^2 + \frac{K_1\eta_1}{2M\eta_2} x_3^2 + \frac{\eta_3^2 K_1}{8M\eta_2\eta_4} x_4^2 + V_1(\underline{\sigma}) \tag{3.42}$$

where $V_1(\underline{\sigma})$ is given by (3.41) with $q = 1$ and $\underline{x} = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix}$.

The time derivative of V along the solution of the system equations is seen to be

$$\dot{V} = -\frac{K_1}{M\eta_2}(\dot{x}_3)^2 - \frac{D}{M} x_2^2 - \frac{\eta_3^2 K_1}{4M\eta_2} x_4^2 + \frac{K_1\eta_3}{M\eta_2} \dot{x}_3 x_4 \tag{3.43}$$

which is seen to be negative semidefinite in the region of interest.

The above Liapunov function V is seen to be different from that given by Siddiqee[12] and is found to give more satisfactory results for the numerical example given in Section 3.4. In fact, Siddiqee's V-function was observed to violate the required sign definite properties

for certain initial operating points, even in a region very close to the origin, thus making it unsuitable for certain operating conditions. On the other hand, the V-function derived above, remains valid for all likely initial operating conditions, in a sufficiently large region, which contains the region of interest.

3.2.4 Equilibrium Points and Stability Region:

At an equilibrium point of the system under discussion, $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$. It can be shown[26] that the solution of the following equations determine the equilibrium points of the system:

$$P_m - \frac{K_1 \eta_3}{\eta_1} E_{ex} \sin \delta - \frac{K_1 \eta_2}{2\eta_1} \sin 2\delta = 0 \tag{3.44}$$

$$E'_q = (\eta_3 E_{ex} + \eta_2 \cos \delta)/\eta_1 \tag{3.45}$$

Solving these nonlinear algebraic equations, the stable prefault equilibrium point is calculated with the corresponding system parameters. Similarly the post fault stable and unstable equilibrium points are computed with the parameters corresponding to the post fault condition. The stability region is found from the inequality

$$V < C \tag{3.46}$$

where C is the value of V evaluated at the unstable post fault equilibrium point. The problem above being that of a single machine, the identification of the stable and

unstable equilibrium points is straightforward. The critical clearing time is computed by numerically integrating the system differential equations (3.13) for the faulted condition, starting from the prefault operating point until the solution trajectory reaches the stability boundary C.

3.2.5 Augmented V-function:

The Liapunov function (3.42), can be made more general by augmenting the quadratic terms as done by Mansour[27]. Thus we can have

$$V = \alpha x_1^2 + \beta x_1 x_2 + \tfrac{1}{2}x_2^2 + \frac{K_1\eta_1}{2M\eta_2} x_3^2 + \frac{\eta_3^2 K_1}{8M\eta_2\eta_4} x_4^2 + V_1(\underline{\sigma}) \tag{3.47}$$

The choice of α and β have to be such that the sign definite properties of V and $\dot{V}$ are not violated in the region of interest. It can be verified that the quadratic form in V will remain nonnegative if $\alpha \geq \beta^2/2$. Additional restrictions have to be imposed by examining $\dot{V}$. It is observed that for this problem, it is not possible to make $\dot{V}$ negative semidefinite in the whole state space with these additional terms present. The objective of introducing these terms is, however, clear. It is to obtain a larger region of stability which is closer to the actual stability region. Therefore, the best one can aim is to select a pair of α and β which will achieve

this objective to the extent possible, without violating the sign definite properties of V and $\dot{V}$. This calls for certain amount of search as evidenced by the numerical example given in Section 3.4.

3.3 GENERALIZED MODEL OF SINGLE-MACHINE SYSTEM

In this section we will consider a single machine-infinite bus system with the synchronous machine modelled to include saliency, flux decay, voltage regulator and governor dynamics. Clearly, we should arrive at a Liapunov function that is most general, of which the previous V-functions must be mere special cases. Indeed it will be shown to be so. Although excellent models with great details are available in the literature for governor representation[61,62] a speed governor with one time constant will be considered here for simplicity[20,27]. Higher order representation does not really pose any problem.

3.3.1 Formulation of the Problem:

The governor action can be approximated by an equivalent first-order system as follows:

$$T_g \dot{P}_m + P_m = P_{mo} - K_g \dot{\delta} \qquad (3.48)$$

where P_m is the instantaneous mechanical power input, and P_{mo} is the post fault steady state value of P_m. K_g is the governor amplification factor and T_g is the equivalent time constant of the governor system.

Define now a state variable

$$x_5 = P_m - P_{mo} \tag{3.49}$$

With the state variables x_1 to x_4 as in Section 3.2.1, the dynamic equations of a single-machine system with saliency, flux decay, voltage regulator and governor dynamics can be written as

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\lambda x_2 + \frac{1}{M} x_5 - \hat{f}_1(\underline{\sigma}) \\
\dot{x}_3 &= -\eta_1 x_3 + \eta_3 x_4 - f_2(\sigma_1) \\
\dot{x}_4 &= -\eta_4 x_4 \\
\dot{x}_5 &= -g\, x_2 - a\, x_5
\end{aligned} \tag{3.50}$$

where $g = K_g/T_g$ and $a = 1/T_g$. Further, in (3.50), the nonlinearity $\hat{f}_1(\underline{\sigma})$ is now given by

$$\hat{f}_1(\underline{\sigma}) = [K_1(\sigma_2+e)\sin(\sigma_1+\delta^s)-K_2\sin 2(\sigma_1+\delta^s)-\hat{P}_m]/M \tag{3.51}$$

where

$$K_2 = \frac{E_B^2\,(x_q-x_d')}{2(x_d'+x_e)(x_q+x_e)} \quad ; \quad \hat{P}_m = K_1\, e \sin\delta^s - K_2 \sin 2\delta^s \tag{3.52}$$

All other symbols have the same meanings as in Section 3.2.1. System (3.50) can now be transformed into the standard form (3.16) by a change of variable[20]

$$\xi = \left(\frac{aD+g}{Ma}\right)x_1 + x_2 + \frac{x_5}{Ma} \tag{3.53}$$

which results in

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} -\lambda & 0 & 0 & 1/M \\ 0 & -\eta_1 & \eta_3 & 0 \\ 0 & 0 & -\eta_4 & 0 \\ -g & 0 & 0 & -a \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\hat{f}}_1(\underline{\sigma}) \\ f_2(\sigma_1) \end{bmatrix}$$

$$\dot{\xi} = -[1 \quad 0] \begin{bmatrix} \hat{f}_1(\underline{\sigma}) \\ f_2(\sigma_1) \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} -Ma/(aD+g) & 0 & 0 & -1/(aD+g) \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} \frac{Ma}{aD+g} \\ 0 \end{bmatrix} \xi \qquad (3.54)$$

For the above system, the transfer function matrix for the linear part obtained by substituting the appropriate matrices and vectors from (3.54) into (3.17), is seen to be

$$W(s) = \begin{bmatrix} \dfrac{(\frac{aD-g}{aD+g})s + \frac{a^2D}{aD+g}}{s(s+\lambda)(s+a)} & 0 \\ 0 & \dfrac{s^2+(a+\lambda)s + (aD+g)/M}{(s+\lambda)(s+\eta_1)(s+a)} \end{bmatrix} \qquad (3.55)$$

Note that the above transfer function reduces to the one derived in Section 3.2.1 when governor action and saliency are neglected.

3.3.2 Derivation of Liapunov Function V:

Following the procedure outlined in 3.2.2 a V-function for the above problem will be derived. As in Section 3.2.3, let

$$V_1 = q \int_0^{\sigma_1} \hat{f}_1(\underline{\sigma})d\sigma_1 \tag{3.56}$$

where $\hat{f}_1(\underline{\sigma})$ is now given by (3.51). On integration we get V_1 as

$$V_1(\underline{\sigma}) = q[\frac{K_1}{M}(\sigma_2+e)(\cos\delta^s-\cos(\sigma_1+\delta^s))-\frac{K_2}{2M}(\cos 2\delta^s-\cos 2(\sigma_1+\delta^s)) - \frac{\hat{P}_m}{M}\sigma_1] \tag{3.57}$$

With $V_1(\underline{\sigma})$ as given by (3.57), the application of the gradient condition (3.20) yields the matrix Q as

$$Q = \begin{bmatrix} q & 0 \\ 0 & qK_1/(M\eta_2) \end{bmatrix} \tag{3.58}$$

Interestingly enough, the matrix Q turns out to be the same as the one in Section 3.2.3. Again for the same reasons discussed in 3.2.3, R is chosen as a zero matrix for this case also. Now with Z(s) defined as in (3.21) and W(s) and Q given by (3.55) and (3.58) respectively, we have:

$$\tfrac{1}{2}[Z(j\omega)+Z^T(-j\omega)] =$$

$$q\begin{bmatrix} \dfrac{k_{112}\omega^2 + k_{110}}{(\omega^2+\lambda^2)(\omega^2+a^2)} & 0 \\ 0 & \dfrac{K_1}{M\eta_2}\left[\dfrac{\omega^6 + k_{224}\omega^4 + k_{222}\omega^2}{(\omega^2+\lambda^2)(\omega^2+\eta_1^2)(\omega^2+a^2)}\right] \end{bmatrix} \tag{3.59}$$

where

$$k_{112} = \frac{\lambda aD - g(a+\lambda)}{aD+g} \quad ; \quad k_{110} = \frac{a^3 D \lambda}{aD+g} \quad ;$$

$$k_{224} = a^2+\lambda^2-g/M \quad ; \quad k_{222} = a^2\lambda^2+\frac{g}{M}(\lambda\eta_1+a\eta_1+a\lambda) \tag{3.60}$$

So as to satisfy the frequency condition (3.22), we shall impose the following two conditions:

$$\lambda\ a\ D - g(a + \lambda) \geq 0$$

$$a^2 + \lambda^2 - g/M \geq 0 \tag{3.61}$$

Now the rest of the construction procedure follows on the lines indicated through (3.25) - (3.28). Factorizing $\frac{1}{2}[Z(s) + Z^T(-s)]$ as given by (3.25), we get $M(s)$ which is seen to be

$$M(s) = \begin{bmatrix} m_{11}(s) & 0 \\ 0 & m_{22}(s) \end{bmatrix}$$

where

$$m_{11}(s) = \sqrt{(qk_{112})}[s+ \sqrt{(k_{110}/k_{112})}]/[(s+\lambda)(s+a)]$$

$$m_{22}(s) = \sqrt{(K_1 q/M\eta_2)}\ s(s+ \sqrt{[\frac{k_{224}}{2}} - \sqrt{(\frac{k_{224}^2-4k_{222}}{4})}])(s + \sqrt{[\frac{k_{224}}{2}} + \sqrt{(\frac{k_{224}^2-4k_{222}}{4})}])/[(s+\lambda)(s+\eta_1)(s+a)]$$

(3.62)

Now with M(s) as given by (3.62), making use of the relation (3.26), the matrix Γ is obtained as

$$\Gamma = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{(\frac{qK_1}{M\eta_2})} \end{bmatrix} \tag{3.63}$$

Assume the 4x2 matrix U to be of the form

$$U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \\ u_{41} & u_{42} \end{bmatrix} \tag{3.64}$$

With this form of U and substituting the appropriate matrices into the identity (3.27), the elements of U are solved for and we get explicit values for the following elements as

$$u_{11} = -\sqrt{(qk_{112})} \quad ; \qquad u_{12} = u_{21} = u_{42} = 0 \ ;$$

$$u_{41} = \sqrt{q} \ (\sqrt{k_{110}} - a \sqrt{k_{112}})/g \quad ;$$

$$u_{22} = \sqrt{(\frac{K_1 q}{M\eta_2})} \ \frac{a\lambda \ \eta_1}{(a \ \lambda + g/M)} \tag{3.65}$$

The remaining elements u_{31} and u_{32} of U can be arbitrarily chosen, depending upon which choice, different L matrix and hence different V-functions will be obtained. For the sake of simplicity however, we will so choose these two elements, as to obtain a diagonal L matrix. This requires the choice of u_{31} and u_{32} to be:

$$u_{31} = 0$$

$$u_{32} = - \frac{\eta_3}{2} \sqrt{(} \ \frac{K_1 q}{M\eta_2})(\ \frac{a\lambda}{a\lambda \ +g/M}) \tag{3.66}$$

Using the above U as given by (3.65) and (3.66) and solving the Liapunov matrix equation (3.28), the L matrix is obtained as

$$L = \mathrm{Diag}(\ell_{ii}) \qquad (i=1,2,3,4)$$

where

$$\ell_{11} = q[\lambda \, aD-g(a+\lambda)]/[2\lambda(aD+g)]$$

$$\ell_{22} = \frac{K \ q \ \eta_1}{2M\eta_2} \ [\frac{a \ \lambda}{a \ \lambda+g/M}]^2$$

$$\ell_{33} = \frac{\eta_3^2 \ K_1 q}{8\eta_2\eta_4 M}(\frac{a \ \lambda}{a\lambda+g/M})^2 \quad ; \qquad \ell_{44} = \frac{q(\sqrt{k_{110}} - a\sqrt{k_{112}})^2}{2g^2 a} \tag{3.67}$$

where k_{110} and k_{112} are as given in (3.60). The Liapunov function for the power system under consideration is given by

$$V(\underline{x},\underline{\sigma}) = \ell_{11}x_2^2 + \ell_{22}x_3^2 + \ell_{33}x_4^2 + \ell_{44}x_5^2 + V_1(\underline{\sigma}) \qquad (3.68)$$

with L given by (3.67) and $V_1(\underline{\sigma})$ as in (3.57). It is easily verified that this Liapunov function reduces to the V-function derived in 3.2.4 when the governor dynamics and saliency are neglected. On the other hand, if we consider only saliency and neglect flux decay, voltage regulator and governor dynamics, the V-function given above reduces to the same as the one derived by Mansour[27] (except for a constant multiplier) by the energy metric algorithm. The V-function for this case as derived from (3.68) differs slightly from the one derived in Section 2.2.3. This can be attributed to the difference in the choice of R and L matrices in this section and the corresponding quantities ρ and ℓ in Section 2.2.3. Nevertheless, this is a perfectly valid V-function. In general therefore, the V-function given in (3.68) is a very general one and if we wish to neglect any of the details that have been included in the above model, we merely set the corresponding state variable as zero and make appropriate changes, in the relevant coefficients. As in Section 3.2.5, the V-function can be made more general by augmenting the quadratic form.

3.4 A NUMERICAL EXAMPLE - MODEL WITH FLUX DECAY AND VOLTAGE REGULATOR

Consider the power system shown in Figure 2.4, but with the following specifications:

$H = 2.76$ MJ/MVA ; $x_d = 1.15$; $x_d' = 0.3$;

The generator rating : 25 MVA ; 60 Hz.

Each of the double circuit transmission line has a reactance of 0.2 p.u. The prefault excitation voltage E_{ex} is taken as 1.22. $T_o' = 6.6$ sec. A symmetrical 3-phase fault is assumed at the middle of one of the parallel transmission lines.

Part (i):

With the aid of the Liapunov function (3.42), the above system is investigated for its transient stability properties with the synchronous machine modelled in the following three ways:

Case (a): Flux decay and voltage regulator action are considered with $T_v = 5$ Sec. and $K_v = 1$.

Case (b): Only flux decay is considered and the voltage regulator action is neglected.

Case (c): Both flux decay and voltage regulator action are neglected.

The critical clearing time for the various cases are estimated for different prefault conditions (This is done by varying P_m) and for various damping coefficients. The results are given in Table 3.1.

Table 3.1: Critical Clearing Time t_c
(T_v = 5 sec. ; K_v = 1.0)

P_m	D	Critical clearing time t_c		
		Case(a)	Case (b)	Case (c)
0.8	0.0	0.36	0.2	0.62
	0.002	0.377	0.22	0.66
	0.004	0.378	0.222	0.72
	0.006	0.38	0.225	0.8
1.0	0.0	0.22	0.12	0.312
	0.002	0.234	0.131	0.32
	0.004	0.237	0.132	0.325
	0.006	0.24	0.133	0.331
1.2	0.0	0.14	0.04	0.201
	0.002	0.152	0.05	0.204
	0.004	0.154	0.051	0.207

The effects of the voltage regulator parameters K_V and T_V on the critical clearing time are studied for Case (a). Typical results are given in Table 3.2 and Table 3.3 respectively.

Part (ii):

Next the augmented Liapunov function given by (3.47) is used to study the case (a) above. In Tables 3.4 - 3.6, the quantity T indicates the time after which the required sign definite property of $\dot{V}$ is violated. Table 3.4 shows the results with β assumed to be zero and α varying. Now keeping α at a fixed value, some positive values of β are tried and the results given in Table 3.5. Finally some negative values for β and positive values for α are used to get the critical clearing time and the computed values tabulated in Table 3.6.

Discussion on the Results:

The numerical example given in Part (i) clearly illustrates the effects of flux decay and voltage regulator action. While flux decay is observed to reduce the critical clearing time, the voltage regulator action tends to neutralize this effect. The degree of neutralization depends on the parameter K_V and T_V although the clearing time is seen to be more sensitive to K_V than to T_V. Increasing the speed of response of the voltage

Table 3.2: Effect of K_v on t_c
(T_v = 5 sec.; D = 0.002)

P_m	K_v	t_c
0.8	0.25	0.26
	0.5	0.3
	0.75	0.34
	1.0	0.377
	1.5	0.4
1.0	0.25	0.16
	0.5	0.18
	0.75	0.22
	1.0	0.234
	1.5	0.243

Table 3.3: Effect of T_v on t_c

(K_v=1; P_m=1; D=0.002)

T_v	t_c
0.5	0.256
1.0	0.2535
1.5	0.2515
2.0	0.249
2.5	0.247
3.0	0.244
3.5	0.242
4.0	0.239

Table 3.4: Variation of t_c with α

($\beta = 0$; $D = 0.002$)

P_m	α	t_c	T
0.8	1.0	0.385	0.72
	2.0	0.393	0.6
	3.0	-	0.28
1.0	1.0	0.239	0.38
	2.0	0.243	0.245
	3.0	-	0.2
1.2	1.0	0.157	0.3
	2.0	0.16	0.2
	3.0	0.165	0.165

Table 3.5: Variation of t_c with β

($\alpha=1$; $D = 0.002$)

β	P_m	t_c	T
0.1336	0.8	0.382	0.66
	1.0	0.23	0.32
	1.2	0.157	0.22
0.5	0.8	0.36	0.6
	1.0	-	0.18
	1.2	-	0.12

Table 3.6: Variation of t_c with α and β

($D = 0.002$)

α	β	P_m	t_c	T
1.75	-0.35	0.8	0.397	0.9
		1.0	0.2475	0.62
		1.2	0.1635	0.52
2.0	-0.35	0.8	0.399	0.94
		1.0	0.248	0.6
		1.2	0.165	0.5
2.25	-0.45	0.8	0.4	0.96
		1.0	0.251	0.62
		1.2	0.168	0.5

regulator does not increase the critical clearing time to a significant degree.

In Part (ii), the general augmented Liapunov function (5.47) is used to analyze the same system. It is seen that by completing the quadratic form to include all the original state variables, it is possible to improve the results. The coefficients associated with these additional terms however, have to be chosen after some search. The coefficients cannot be fixed by having an eye merely on the V-function. Their effect on $\dot{V}$ also has to be kept in view. A large number of combinations of α and β were tried. It was observed that very small values of these coefficients did not affect the sign definite properties of V and $\dot{V}$ significantly, but then the improvement achieved in the critical clearing time t_c was too small. On the other hand, very large values of α and β were seen to render both V and $\dot{V}$ indefinite in a region too close to the post fault equilibrium point. In the numerical example above a positive β did not improve the results. Instead it made the results worse compared to that with zero value of β. On the other hand, a negative value for β gives more accurate value of t_c, provided it is chosen judiciously. Perhaps the best way to arrive at a proper pair of α and β would seem to be to first choose a value of α keeping β as zero, and then keeping this value of α fixed, a search

is made for β. Clearly, the augmented Liapunov function will require more number of computation due to the search process required to fix the coefficients α and β. Whether the additional effort and computational time involved are worth or not has to be assessed in terms of the improved values of t_c obtained. Using this general Liapunov function also, parameter analysis can be carried out.

3.5 CONCLUSION

A systematic procedure for constructing a Liapunov function for a single machine connected to an infinite bus, with flux decay effect and voltage regulator action included, has been developed in the first part of the chapter. A more complete model of the synchronous machine to include transient saliency and governor dynamics also, is taken up next and a generalized Liapunov function derived. The V-function derived earlier, is seen to be a particular case of this generalized Liapunov function. An augmented Liapunov function obtained by completing the quadratic form is also discussed. The numerical example illustrates the ease with which the various aspects of the transient stability problem of systems with greater details of modelling, can be investigated. Needless to mention that generalized Liapunov function can be utilized for a complete parameter analysis to include the parameters of the governor also.

CHAPTER IV

MULTIMACHINE SYSTEM WITH FLUX DECAY EFFECT

4.1 INTRODUCTION

This chapter is concerned with the formulation of the multimachine problems to include flux decay effect, in a form suitable for stability analysis through the Liapunov approach and development of suitable V-functions. Although multimachine systems with simple machine model (a constant voltage behind a transient reactance) have been popular in earlier studies, extension to incorporate machine models with greater details are desirable. It would indeed be desirable to include all the refinements discussed in Section 3.3, in a multimachine system. Unfortunately, the problem then becomes so complex, that it is practically impossible to handle with the techniques known at the present time. We will deal with a two-machine system first, and then a three-machine system, with flux decay effects considered. The results of these two cases would then be extended to a general k-machine system. It is first of all noted that as in the case of the single machine system considered in Chapter III, the nonlinearities are of the multiargument type. It is evident that Kalman's construction procedure cannot be used to derive a V-function. An attempt was made to use the modified

version of Kalman's construction procedure developed in Chapter III, but without success. One therefore has to fall back on other techniques to generate a V-function. A number of techniques such as variable gradient method[10,13], Cartwright method[18], Aizerman's method[18], Zubov's method[11,26], method of integration by parts and method of Infante and Clark[13] have been used earlier on simpler situations. In the present work, a version of'integration by parts' method[13,30] is used to develop appropriate Liapunov functions.

4.2 TWO-MACHINE SYSTEM

Formulation of the Problem:

Consider two synchronous machines connected together through a transmission line. Neglecting resistances and transient saliency, and assuming constant input power and constant exciter voltage the equations of motion of the two-machines can be written as

$$\begin{aligned} M_1 \ddot{\delta}_1 + D_1 \dot{\delta}_1 &= P_{m1} - E_1E_2 Y_{12} \sin(\delta_1-\delta_2) \\ M_2 \ddot{\delta}_2 + D_2 \dot{\delta}_2 &= P_{m2} - E_1E_2 Y_{12} \sin(\delta_2-\delta_1) \qquad (4.1) \end{aligned}$$

where E_1 and E_2 are now the voltages back of transient reactances for the machines 1 and 2 respectively and are assumed to be proportional to the field flux linkages. Y_{12} is the short circuit transfer susceptance between the internal buses representing the voltages E_1

and E_2. The rates of changes of E_1 and E_2 due to flux decay effect can now be written as:

$$\dot{E}_i = (E_{exi} - E_{Ii})/T'_{oi} \qquad (i=1,2) \qquad (4.2)$$

where E_{exi} is the exciter voltage referred to the armature circuit and E_{Ii} is a voltage corresponding to field current for the ith machine. Working on similar lines as in Crary[44,2] it can be shown that

$$E_{Ii} = [1+(x_{di}-x'_{di})/X_{ii}]E_i-(x_{di}-x'_{di})\frac{E_j}{X_{ij}}\cos\delta_{ij}$$

$$(i,j = 1,2 \; ; \quad j \neq i) \qquad (4.3)$$

where X_{ii} and X_{ij} are the driving point and transfer reactances with transient reactances. On substitution of (4.3) into (4.2), we have:

$$\dot{E}_i = E_{exi}/T'_{oi}-a_{ii}E_i + a_{ij}E_j \cos\delta_{ij} \quad (i,j = 1,2; \; j\neq i)$$

$$(4.4)$$

where

$$a_{ii} = (X_{ii} + x_{di} - x'_{di})/(X_{ii} \; T'_{oi})$$

$$a_{ij} = (x_{di}-x'_{di})/(X_{ij} \; T'_{oi}) \qquad (4.5)$$

Let e_1 and e_2 be the post fault steady state values of E_1 and E_2 respectively and δ_1^s and δ_2^s be the post fault steady state values of δ_1 and δ_2 respectively. It is easily seen that

$$P_{m1} = e_1 e_2 \, Y_{12} \sin \delta_{12}^s = -P_{m2}$$

where Y_{12} corresponds to the post fault condition.

Define the state variables:

$$\begin{aligned}
x_1 &= \delta_1 - \delta_1^s \\
x_2 &= \delta_2 - \delta_2^s \\
x_3 &= \dot{x}_1 \\
x_4 &= \dot{x}_2 \\
x_5 &= E_1 - e_1 \\
x_6 &= E_2 - e_2
\end{aligned} \tag{4.6}$$

With (4.6) substituted, (4.4) on simplification, can be written as:

$$\begin{aligned}
\dot{E}_1 &= -a_{11}x_5 - a_{12}[e_2 \cos\delta_{12}^s - (x_6 + e_2) \cos\delta_{12}] \\
\dot{E}_2 &= -a_{22}x_6 - a_{21}[e_1 \cos\delta_{21}^s - (x_5 + e_1) \cos\delta_{21}]
\end{aligned} \tag{4.7}$$

Now define

$$\begin{aligned}
\sigma_1 &= x_1 - x_2 \\
\sigma_2 &= x_5 \\
\sigma_3 &= x_6
\end{aligned} \tag{4.8}$$

and also

$$\begin{aligned}
f_1(\underline{\sigma}) &= (\sigma_2 + e_1)(\sigma_3 + e_2)Y_{12} \sin(\sigma_1 + \delta_{12}^s) - P_{m1} \\
f_2(\underline{\sigma}) &= a_{12}[e_2 \cos \delta_{12}^s - (\sigma_3 + e_2) \cos(\sigma_1 + \delta_{12}^s)] \\
f_3(\underline{\sigma}) &= a_{21}[e_1 \cos \delta_{12}^s - (\sigma_2 + e_1) \cos(\sigma_1 + \delta_{12}^s)]
\end{aligned} \tag{4.9}$$

With (4.9), (4.8) and (4.6), the complete system differential equations (4.1) and (4.7) can be put in the form:

$$
\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= -\lambda_1 x_3 - f_1(\underline{\sigma})/M_1 \\
\dot{x}_4 &= -\lambda_2 x_4 + f_1(\underline{\sigma})/M_2 \\
\dot{x}_5 &= -a_{11} x_5 - f_2(\underline{\sigma}) \\
\dot{x}_6 &= -a_{22} x_6 - f_3(\underline{\sigma})
\end{aligned}
\tag{4.10}
$$

Let us introduce a change of variables now, defined by:

$$
\begin{aligned}
\xi_1 &= D_1 x_1 + M_1 x_3 \\
\xi_2 &= D_2 x_2 + M_2 x_4
\end{aligned}
\tag{4.11}
$$

With these variables replacing x_1 and x_2, the system above can be transformed into the standard form:

$$
\begin{aligned}
\dot{\underline{x}} &= F \underline{x} - G \underline{f}(\underline{\sigma}) \\
\dot{\underline{\xi}} &= -N \underline{f}(\underline{\sigma}) \\
\underline{\sigma} &= H^T \underline{x} + P \underline{\xi}
\end{aligned}
\tag{4.12}
$$

where

$$
\underline{x} = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} ; \quad \underline{f}(\underline{\sigma}) = \begin{bmatrix} f_1(\underline{\sigma}) \\ f_2(\underline{\sigma}) \\ f_3(\underline{\sigma}) \end{bmatrix} ; \quad \underline{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} ; \quad \underline{\sigma} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}
$$

$$F = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 \\ 0 & -\lambda_2 & 0 & 0 \\ 0 & 0 & -a_{11} & 0 \\ 0 & 0 & 0 & -a_{22} \end{bmatrix} ; \quad G = \begin{bmatrix} 1/M_1 & 0 & 0 \\ -1/M_2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ;$$

$$N = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} ;$$

$$H^T = \begin{bmatrix} -M_1/D_1 & M_2/D_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad P = \begin{bmatrix} 1/D_1 & -1/D_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{4.13}$$

Liapunov Function for the Two-Machine System:

Assume tentatively a Liapunov function of the form

$$V_o(\underline{x}, \underline{\sigma}) = \ell_{11} x_3^2 + \ell_{22} x_4^2 + \ell_{33} x_5^2 + \ell_{44} x_6^2 + q \int_0^{\sigma_1} f_1(\underline{\sigma}) \, d\sigma_1 \tag{4.14}$$

where q is yet to be determined. The inclusion of integrals of other nonlinearities was found to give rise to some difficulties in ensuring the sign definite property of the time derivative of the Liapunov function. The positive definite property of V_o can be established along the lines similar to the single-machine system.

Now taking the time derivative along the system equations (4.12) and on some simplification, we have

$$\dot{V}_o = -2\ell_{11}\lambda_1 x_3^2 - 2\ell_{22}\lambda_2 x_4^2 - 2\ell_{11} f_1 x_3/M_1 + 2\ell_{22} f_1 x_4/M_2$$
$$+ 2\ell_{33} x_5 \dot{x}_5 + 2\ell_{44} x_6 \dot{x}_6 + q f_1 x_3 - q f_1 x_4$$
$$+ q Y_{12} (\sigma_3 + e_2)[\cos \delta_{12}^s - \cos(\sigma_1 + \delta_{12}^s)] \dot{x}_5$$
$$+ q Y_{12}(\sigma_2 + e_1)[\cos\delta_{12}^s - \cos(\sigma_1 + \delta_{12}^s)] \dot{x}_6 \qquad (4.15)$$

Choose

$$\ell_{11} = M_1 q/2 \quad ; \qquad \ell_{22} = M_2 q/2 \qquad (4.16)$$

With (4.16) substituted in (4.15) and on some rearrangement, we have

$$\dot{V}_o = -2\ell_{11}\lambda_1 x_3^2 - 2\ell_{22}\lambda_2 x_4^2$$
$$- \frac{2\ell_{33}}{a_{11}} \dot{x}_5[-a_{11}x_5 - \frac{qY_{12}a_{11}}{2\ell_{33}} (e_2\cos\delta_{12}^s - (\sigma_3+e_2)\cos(\sigma_1+\delta_{12}^s))]$$
$$- \frac{2\ell_{44}}{a_{22}} \dot{x}_6[-a_{22}x_6 - \frac{qY_{12}a_{22}}{2\ell_{44}}(e_1\cos\delta_{12}^s - (\sigma_2+e_1)\cos(\sigma_1+\delta_{12}^s))]$$
$$+ (qY_{12}\cos\delta_{12}^s)x_6\dot{x}_5 + (qY_{12}\cos\delta_{12}^s)x_5\dot{x}_6 \qquad (4.17)$$

Now choose

$$\ell_{33} = \frac{q\, Y_{12}\, a_{11}}{2a_{12}} \; ; \qquad \ell_{44} = \frac{q\, Y_{12}\, a_{22}}{2a_{21}} \qquad (4.18)$$

Define

$$\alpha = q\, Y_{12} \cos \delta_{12}^{s} \tag{4.19}$$

With (4.19) and (4.18) substituted, (4.17) now becomes

$$\dot{V}_o = -2\ell_{11}\lambda_1 x_3^2 - 2\ell_{22}\lambda_2 x_4^2 - 2\ell_{33}(\dot{x}_5)^2/a_{11} - 2\ell_{44}(\dot{x}_6)^2/a_{22} + \alpha(\dot{x}_5 x_6 + x_5 \dot{x}_6) \tag{4.20}$$

Since the last term can make $\dot{V}_o$ indefinite, it must be eliminated. Let

$$\dot{v}_1 = \alpha(\dot{x}_5 x_6 + x_5 \dot{x}_6)$$

Integrating this with respect to time, we get

$$v_1 = \alpha\, x_5\, x_6$$

where with no loss of generality, the constant of integration is taken to be zero. If we now choose a Liapunov function V given by $V = V_o - v_1$, we have a V-function which has a negative semidefinite time derivative. Thus the modified and final Liapunov function is

$$V = \ell_{11} x_3^2 + \ell_{22} x_4^2 + \ell_{33} x_5^2 + \ell_{44} x_6^2 - \alpha\, x_5\, x_6 + q\int_0^{\sigma_1} f_1(\underline{\sigma})\, d\,\sigma_1 \tag{4.21}$$

with the time derivative

$$\dot{V} = -2\ell_{11}\lambda_1 x_3^2 - 2\ell_{22}\lambda_2 x_4^2 - 2\ell_{33}(\dot{x}_5)^2/a_{11} - 2\ell_{44}(\dot{x}_6)^2/a_{22} \tag{4.22}$$

It is now necessary to ensure that V itself remains positive definite. This can be guaranteed if the quadratic form in (4.21) remains positive definite in $\underline{x}$, which requires the condition that

$$\ell_{33}\ell_{44} - \alpha^2/4 > 0 \qquad (4.23)$$

The coefficients ℓ_{33} and ℓ_{44} when simplified, turn out to be

$$\ell_{33} = \frac{q}{2}\,(X_{11} + x_{d1} - x'_{d1})/[X_{11}(x_{d1} - x'_{d1})]$$

and

$$\ell_{44} = \frac{q}{2}(X_{22} + x_{d2} - x'_{d2})/[\,X_{22}(x_{d2} - x'_{d2})\,]$$

With these values of ℓ_{33} and ℓ_{44} and with α as given by (4.19), the condition (4.23) will be valid for a practical system and thus V will remain positive definite.

4.3 THREE-MACHINE SYSTEM

Formulation:

Consider now a three-machine system, wherein three synchronous machines are interconnected through transmission lines. With the same assumptions as in the two-machine system, the equations of motion of the rotors of the three machines can be written as

$$M_i\ddot{\delta}_i + D_i\dot{\delta}_i = P_{mi} - \sum_{\substack{j=1\\ j\neq i}}^{3} E_iE_jY_{ij}\sin(\delta_i-\delta_j) \qquad (4.24)$$

$$(i = 1,2,3)$$

The rates of changes of the voltages E_1, E_2 and E_3 due to the variation of flux linkages can be derived by extending the procedure followed in the two-machine system and we will get:

$$\dot{E}_i = \frac{E_{exi}}{T'_{oi}} - a_{ii} E_i + \sum_{\substack{j=1 \\ j \neq i}}^{3} a_{ij} E_j \cos \delta_{ij} \quad (i=1,2,3) \qquad (4.25)$$

where

$$a_{ii} = \frac{1}{T'_{oi}} [1 + \frac{x_{di} - x'_{di}}{X_{ii}}] \quad \text{and} \quad a_{ij} = \frac{(x_{di} - x'_{di})}{X_{ij} T'_{oi}} \qquad (4.26)$$

As before let e_i and δ_i^s $(i = 1,2,3)$ be the post fault steady state values of E_i and δ_i $(i = 1,2,3)$ respectively. Define

$$x_i = \delta_i - \delta_i^s$$

$$x_{(3+i)} = \dot{x}_i$$

$$x_{(6+i)} = E_i - e_i \qquad (i = 1,2,3) \qquad (4.27)$$

With (4.27) substituted into (4.25), we get, on following the same line of simplification as in the two-machine system,

$$\dot{x}_7 = -a_{11}x_7 - a_{12}[e_2 \cos \delta_{12}^s -(x_8 + e_2) \cos \delta_{12}]$$

$$- a_{13}[e_3 \cos \delta_{13}^s - (x_9 + e_3) \cos \delta_{13}]$$

$$\dot{x}_8 = -a_{22}\, x_8 - a_{21}[e_1 \cos \delta^s_{21} - (x_7 + e_1) \cos \delta_{21}]$$

$$-a_{23}[e_3 \cos \delta^s_{23} - (x_9 + e_3) \cos \delta_{23}]$$

$$\dot{x}_9 = -a_{33}\, x_9 - a_{31}[e_1 \cos \delta^s_{31} - (x_7 + e_1) \cos \delta_{31}]$$

$$-a_{32}\, [e_2 \cos \delta^s_{32} - (x_8 + e_2) \cos \delta_{32}] \qquad (4.28)$$

Now define

$$\sigma_1 = x_1 - x_2\ ; \quad \sigma_2 = x_1 - x_3\ ; \quad \sigma_3 = x_2 - x_3\ ;$$

$$\sigma_4 = x_7\ ; \quad \sigma_5 = x_8\ ; \quad \sigma_6 = x_9 \qquad (4.29)$$

Also define the nonlinearities

$$f_1(\underline{\sigma}) = (\sigma_4+e_1)(\sigma_5+e_2)Y_{12} \sin(\sigma_1+\delta^s_{12}) - e_1 e_2 Y_{12} \sin \delta^s_{12}$$

$$f_2(\underline{\sigma}) = (\sigma_4+e_1)(\sigma_6+e_3)Y_{13} \sin(\sigma_2+\delta^s_{13}) - e_1 e_3 Y_{13} \sin \delta^s_{13}$$

$$f_3(\underline{\sigma}) = (\sigma_5+e_2)(\sigma_6+e_3)Y_{23} \sin(\sigma_3+\delta^s_{23}) - e_2 e_3 Y_{23} \sin \delta^s_{23}$$

$$f_4(\underline{\sigma}) = a_{12}[e_2 \cos\delta^s_{12} - (\sigma_5+e_2)\cos(\sigma_1+\delta^s_{12})]$$

$$+ a_{13}[e_3 \cos\delta^s_{13} - (\sigma_6+e_3)\cos(\sigma_2+\delta^s_{13})]$$

$$f_5(\underline{\sigma}) = a_{21}[e_1 \cos\delta^s_{12} - (\sigma_4+e_1)\cos(\sigma_1+\delta^s_{12})]$$

$$+ a_{23}[e_3 \cos\delta^s_{23} - (\sigma_6+e_3)\cos(\sigma_3+\delta^s_{23})]$$

$$f_6(\underline{\sigma}) = a_{31}[e_1 \cos\delta^s_{13} - (\sigma_4+e_1)\cos(\sigma_2+\delta^s_{13})]$$

$$+a_{32}\, [e_2 \cos \delta^s_{23} - (\sigma_5 + e_2)\cos(\sigma_3+\delta^s_{23})] \qquad (4.30)$$

Making use of (4.27), (4.29) and (4.30), the system differential equations (4.24) and (4.28) can be put in the form:

$$\dot{x}_1 = x_4$$

$$\dot{x}_2 = x_5$$

$$\dot{x}_3 = x_6$$

$$\dot{x}_4 = -\lambda_1\, x_4 - f_1/M_1 - f_2/M_1$$

$$\dot{x}_5 = -\lambda_2\, x_5 + f_1/M_2 - f_3/M_2$$

$$\dot{x}_6 = -\lambda_3\, x_6 + f_2/M_3 + f_3/M_3$$

$$\dot{x}_7 = -\, a_{11}\, x_7 - f_4$$

$$\dot{x}_8 = -\, a_{22}\, x_8 - f_5$$

$$\dot{x}_9 = -a_{33}\, x_9 - f_6 \qquad (4.31)$$

(Note that $f_i(\underline{\sigma})$ is written here as f_i $(i=1,2,\ldots,6)$ for brevity).

Define new state variables

$$\xi_1 = D_1\, x_1 + M_1\, x_4$$

$$\xi_2 = D_2\, x_2 + M_2\, x_5$$

$$\xi_3 = D_3\, x_3 + M_3\, x_5 \qquad (4.32)$$

With these new variables replacing x_1, x_2 and x_3, the system can now be put in the standard form (4.12) with the vectors and matrices given by

$$\underline{x} = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} ; \quad \underline{f}(\underline{\sigma}) = \begin{bmatrix} f_1(\underline{\sigma}) \\ f_2(\underline{\sigma}) \\ f_3(\underline{\sigma}) \\ f_4(\underline{\sigma}) \\ f_5(\underline{\sigma}) \\ f_6(\underline{\sigma}) \end{bmatrix} ; \quad \underline{\sigma} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} \qquad \underline{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}$$

$$F = -\mathrm{diag}(\lambda_1, \lambda_2, \lambda_3, a_{11}, a_{22}, a_{33})$$

$$G = \begin{bmatrix} 1/M_1 & 1/M_1 & 0 & 0 & 0 & 0 \\ -1/M_2 & 0 & 1/M_2 & 0 & 0 & 0 \\ 0 & -1/M_3 & -1/M_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$H^T = \begin{bmatrix} -M_1/D_1 & M_2/D_2 & 0 & 0 & 0 & 0 \\ -M_1/D_1 & 0 & M_3/D_3 & 0 & 0 & 0 \\ 0 & -M_2/D_2 & M_3/D_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1/D_1 & -1/D_2 & 0 \\ 1/D_1 & 0 & -1/D_3 \\ 0 & 1/D_2 & -1/D_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (4.33)$$

Liapunov Function for the 3-Machine System:

Tentatively let us assume a Liapunov function V_o as

$$V_o = \ell_{11}x_4^2 + \ell_{22}x_5^2 + \ell_{33}x_6^2 + \ell_{44}x_7^2 + \ell_{55}x_8^2 + \ell_{66}x_9^2 + \sum_{i=1}^{3} q_i \int_o^{\sigma_i} f_i(\underline{\sigma})\, d\sigma_i \qquad (4.34)$$

Eventhough we have a term in (4.34) which is the sum of integrals of nonlinearities which are of 'multi-argument' type, the behaviour of V_o around the origin

will be similar to that of the single machine case. Thus it will be positive definite in a finite neighbourhood of the origin. Differentiating (4.34) along the system equations in the standard form and assuming

$$q_1 = q_2 = q_3 = q \;;$$

$$\ell_{11} = \frac{q}{2} M_1 \;;\; \ell_{22} = qM_2/2 \;;\; \ell_{33} = qM_3/2 \qquad (4.35)$$

we have

$$\dot{V}_o = -2\ell_{11}\lambda_1 x_4^2 - 2\ell_{22}\lambda_2 x_5^2 - 2\ell_{33}\lambda_3 x_6^2$$

$$-2\dot{x}_7 \frac{\ell_{44}}{a_{11}}[-a_{11}x_7 - \frac{q_1 Y_{12} a_{11}}{2\ell_{44}}(e_2 \cos\delta_{12}^s -$$

$$(\sigma_5+e_2)\cos(\sigma_1+\delta_{12}^s)) - \frac{qY_{13}a_{11}}{2\ell_{44}}(e_3 \cos\delta_{13}^s -$$

$$(\sigma_6+e_3)\cos(\sigma_2+\delta_{13}^s))] + (qY_{12}\cos\delta_{12}^s)x_8\dot{x}_7 +$$

$$(q_2 Y_{13}\cos\delta_{13}^s)x_9\dot{x}_7 - 2\dot{x}_8 \frac{\ell_{55}}{a_{22}}[-a_{22}x_8 -$$

$$\frac{qY_{12}a_{22}}{2\ell_{55}}(e_1\cos\delta_{12}^s - (\sigma_4+e_1)\cos(\sigma_1+\delta_{12}^s)) - \frac{qY_{23}a_{22}}{2\ell_{55}}(e_3\cos\delta_{23}^s -$$

$$(\sigma_6+e_3)\cos(\sigma_3+\delta_{23}^s))] + (qY_{12}\cos\delta_{12}^s)x_7\dot{x}_8 + (qY_{23}\cos\delta_{23}^s)x_9\dot{x}_8$$

$$-2\dot{x}_9 \frac{\ell_{66}}{a_{33}}[-a_{33}x_9 - \frac{qY_{13}a_{33}}{2\ell_{66}}(e_1\cos\delta_{13}^s - (\sigma_4+e_1)\cos(\sigma_2+\delta_{13}^s)) -$$

$$\frac{qY_{23}a_{33}}{2\ell_{66}}(e_2\cos\delta_{23}^s - (\sigma_5+e_2)\cos(\sigma_3+\delta_{23}^s))] + (qY_{13}\cos\delta_{13}^s)x_7\dot{x}_9$$

$$+ (qY_{23}\cos\delta_{23}^s)x_8\dot{x}_9 \qquad (4.36)$$

Now choose ℓ_{44}, ℓ_{55} and ℓ_{66} such that

$$\frac{qY_{12}a_{11}}{2\ell_{44}} = a_{12} \; ; \quad \frac{qY_{13}a_{11}}{2\ell_{44}} = a_{13} \; ; \quad \frac{qY_{12}a_{22}}{2\ell_{55}} = a_{21} \; ;$$

$$\frac{qY_{23}a_{22}}{2\ell_{55}} = a_{23} \; ; \quad \frac{qY_{13}a_{33}}{2\ell_{66}} = a_{31} \; ; \quad \frac{qY_{23}a_{33}}{2\ell_{66}} = a_{32} \tag{4.37}$$

Also define

$$\begin{aligned} \alpha_1 &= q\, Y_{12} \cos \delta_{12}^{s} \\ \alpha_2 &= q\, Y_{13} \cos \delta_{13}^{s} \\ \alpha_3 &= q\, Y_{23} \cos \delta_{23}^{s} \end{aligned} \tag{4.38}$$

With (4.37) and (4.38), the expression for $\dot{V}_o$ reduces to

$$\begin{aligned} \dot{V}_o = {} & -2\ell_{11}\lambda_1 x_4^2 - 2\ell_{22}\lambda_2 x_5^2 - 2\ell_{33}\lambda_3 x_6^2 - \frac{2\ell_{44}}{a_{11}}(\dot{x}_7)^2 - \\ & \frac{2\ell_{55}}{a_{22}}(\dot{x}_8)^2 - \frac{2\ell_{66}}{a_{33}}(\dot{x}_4)^2 + \alpha_1(\dot{x}_7 x_8 + x_7\dot{x}_8) + \\ & \alpha_2(\dot{x}_7 x_9 + \dot{x}_9 x_7) + \alpha_3(\dot{x}_8 x_9 + x_8\dot{x}_9) \end{aligned} \tag{4.39}$$

By the same reasoning as in the two-machine problem seen earlier in this chapter, we will ensure the negative semidefinite property of the derivative of the final Liapunov function chosen, by eliminating from (4.39) those terms that may make $\dot{V}_o$ indefinite. Thus define

$$v = \alpha_1 x_7 x_8 + \alpha_2 x_7 x_9 + \alpha_3 x_8 x_9 \tag{4.40}$$

Now subtracting this v from V_o we get the modified and final Liapunov function as

$$V = \ell_{11}x_4^2 + \ell_{22}x_5^2 + \ell_{33}x_6^2 + \ell_{44}x_7^2 + \ell_{55}x_8^2 + \ell_{66}x_9^2 - \alpha_1 x_7 x_8 - \alpha_2 x_7 x_9 - \alpha_3 x_8 x_9 + q \sum_{i=1}^{3} \int_o^{\sigma_i} f_i(\underline{\sigma}) d\sigma_i \tag{4.41}$$

with its time derivative,

$$\dot{V} = -2\ell_{11}\lambda_1 x_4^2 - 2\ell_{22}\lambda_2 x_5^2 - 2\ell_{33}\lambda_3 x_6^2 - \frac{2\ell_{44}}{a_{11}}(\dot{x}_7)^2 - \frac{2\ell_{55}}{a_{22}}(\dot{x}_8)^2 - \frac{2\ell_{66}}{a_{33}}(\dot{x}_9)^2 \tag{4.42}$$

It can be shown on appropriate substitution and simplification, that

$$\ell_{44} = \frac{q}{2}\frac{(X_{11} + x_{d1} - x'_{d1})}{X_{11}(x_{d1} - x'_{d1})} ; \quad \ell_{55} = \frac{q}{2}\frac{(X_{22} + x_{d2} - x'_{d2})}{X_{22}(x_{d2} - x'_{d2})} ;$$

$$\ell_{66} = \frac{q}{2}\frac{(X_{33} + x_{d3} - x'_{d3})}{X_{33}(x_{d3} - x'_{d3})} \tag{4.43}$$

Clearly $\dot{V}$ is negative semidefinite. Without loss of generality q can be chosen as unity. The positive definiteness of V can be guaranteed if the quadratic form in (4.41) is positive definite which can be ensured if the following three conditions derived by applying Sylvester's criterion, are satisfied:

(i) $1/C_1 - \cos^2\delta_{12}^s > 0$

(ii) $1/C_3 - \cos^2\delta_{23}^s > 0$

(iii) $1-(C_1\cos^2\delta_{12}^s+C_2\cos^2\delta_{13}^s+C_3\cos^2\delta_{23}^s +$

$$C_4\cos\delta_{12}^s\cos\delta_{13}^s\cos\delta_{23}^s) > 0 \tag{4.44}$$

where

$$C_1 = \frac{(x_{d1} - x'_{d1})(x_{d2} - x'_{d2})\ X_{11}\ X_{22}}{X_{12}^2(X_{11}+x_{d1}-x'_{d1})(X_{22}+x_{d2}-x'_{d2})}$$

$$C_2 = \frac{(x_{d1} - x'_{d1})\ (x_{d3} - x'_{d3})\ X_{11}\ X_{33}}{X_{13}^2(X_{11}+x_{d1}-x'_{d1})(X_{33}+x_{d3}-x'_{d3})}$$

$$C_3 = \frac{(x_{d2} - x'_{d2})\ (x_{d3}-x'_{d3})\ X_{22}\ X_{33}}{X_{23}^2(X_{22}+x_{d2}-x'_{d2})(X_{33}+x_{d3}-x'_{d3})}$$

$$C_4 = \frac{2(x_{d1}-x'_{d1})(x_{d2} - x'_{d2})(x_{d3} - x'_{d3})\ X_{11}\ X_{22}\ X_{33}}{X_{12}X_{13}X_{23}(X_{11}+x_{d1}-x'_{d1})(X_{22}+x_{d2}-x'_{d2})(X_{33}+x_{d3}-x'_{d3})} \tag{4.45}$$

For a practical system, the conditions (4.44) are always satisfied, hence ensuring the overall positive definiteness of V in the region of interest.

Illustrative Example:

Consider the system shown in Figure 4.1. Assume the values of the various reactances as follows (all in p.u.):

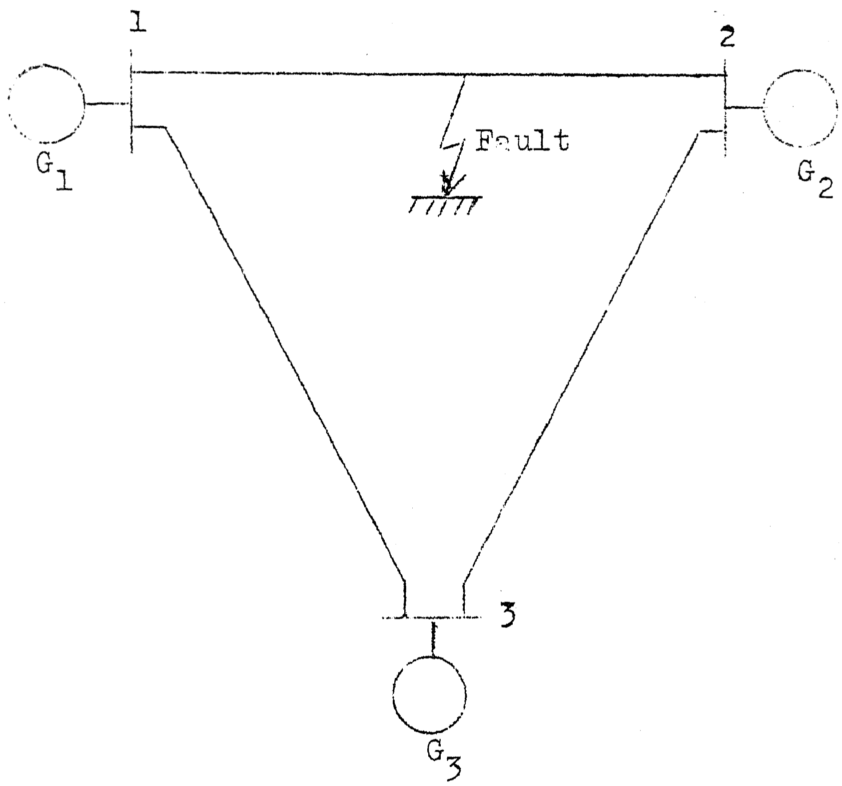

FIG.4.1 A THREE-MACHINE SYSTEM.

$x_{d1} = 1.1$; $x_{d2} = 1.15$; $x_{d3} = 1.2$;

$x'_{d1} = 0.23$; $x'_{d2} = 0.35$; $x'_{d3} = 0.25$

The transmission line reactances between the ith and jth buses denoted by x_{eij} are:

$x_{e12} = 0.2$; $x_{e13} = 0.4$; $x_{e23} = 0.5$

Assume a fault to occur along the transmission line 1-2 which is cleared by opening the line 1-2. For such a system, the reactances involved for computing the coefficients in the Liapunov function can be verified to be

$X_{12} = 3.62$; $X_{13} = 1.065$; $X_{23} = 1.438$

$X_{11} = 0.822$; $X_{22} = 1.025$; $X_{33} = 0.61$

These values result in (with $q = 1$).

$\ell_{44} = 1.19$; $\ell_{55} = 1.11$ and $\ell_{66} = 1.35$

$\alpha_1 = 0.277\cos\delta^s_{12}$; $\alpha_2 = 0.94\cos\delta^s_{13}$;

$\alpha_3 = 0.697\cos\delta^s_{23}$.

Thus the Liapunov function for this system is given by

$$V(\underline{x},\underline{\sigma}) = \frac{M_1}{2}x_4^2 + \frac{M_2}{2}x_5^2 + \frac{M_3}{2}x_6^2 + 1.19x_7^2 + 1.11x_8^2 +$$

$$1.35x_9^2 - 0.277x_7x_8\cos\delta^s_{12} - 0.94x_7x_9\cos\delta^s_{13} -$$

$$0.697x_8x_9\cos\delta^s_{23} + \sum_{i=1}^{3} \int_0^{\sigma_i} f_i(\underline{\sigma})d\sigma_i$$

It can be verified that the quadratic part is always positive. This can be shown by directly applying Sylvester's criterion to the above Liapunov function or equivalently by applying conditions (4.44) wherein we have

$$C_1 = 0.0145 \; ; \; C_2 = 0.1375 \; ; \; C_3 = 0.081 \; ;$$
$$C_4 = 0.0256.$$

Thus the above V-function will be positive definite in a finite neighbourhood of the origin. Beyond this region, V may become negative due to the contribution of the integral terms. The region of interest, however will lie within this neighbourhood.

Several values of the external reactances x_{eij}, spread over a wide range, were tried and it was observed that the conditions (4.44) were always satisfied. Thus the V-function developed here, seems to be a perfectly valid Liapunov function, which can be used for the study of all aspects of stability.

4.4 EXTENSION TO k-MACHINE SYSTEM

Formulation of the Problem:

On close observation of the two-machine and three-machine system formulations, an extension to a k-machine system can be made by induction.

The differential equations governing the rotor motions and the variation of the voltages proportional to the field flux linkages for the k-machine system are given respectively by

$$M_i\ddot{\delta}_i+D_i\dot{\delta}_i = P_{mi} - \sum_{\substack{j=1 \\ j\neq i}}^{k} E_iE_jY_{ij}\sin(\delta_i-\delta_j)$$
$$(i=1,2,\ldots,k) \qquad (4.46)$$

$$\dot{E}_i = \frac{1}{T'_{oi}}[E_{exi}-E_i(1+(x_{di}-x'_{di})/X_{ii})+(x_{di}-x'_{di})\sum_{\substack{j=1 \\ j\neq i}}^{k} \frac{E_j}{X_{ij}}\cos\delta_{ij}]$$
$$(i=1,2,\ldots,k) \qquad (4.47)$$

Define now a 3k-dimensional vector $\hat{\underline{X}}$ as

$$\hat{\underline{X}} = \begin{bmatrix} \underline{X}_1 \\ \underline{X}_2 \\ \underline{X}_3 \end{bmatrix} \qquad (4.48)$$

where the k-vectors $\underline{X}_1$, $\underline{X}_2$ and $\underline{X}_3$ are defined by

$$\underline{X}_1 = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} = \underline{\delta}-\underline{\delta}^s; \quad \underline{X}_2 = \begin{bmatrix} x_{k+1} \\ x_{k+2} \\ \vdots \\ x_{2k} \end{bmatrix} = \dot{\underline{X}}_1 ;$$

$$\underline{X}_3 = \begin{bmatrix} x_{2k+1} \\ x_{2k+2} \\ \vdots \\ x_{3k} \end{bmatrix} = \underline{E} - \underline{e} \qquad (4.49)$$

Here $\underline{\delta}$ and $\underline{E}$ are column vectors with components $(\delta_1, \delta_2, \ldots, \delta_k)$ and $(E_1, E_2, \ldots, E_k)$ respectively. $\underline{\delta}^s$ and $\underline{e}$ are the post fault steady state values of $\underline{\delta}$ and $\underline{E}$ respectively. With the aid of (4.49), the equation (4.47) can be rewritten just as in the two and three-machine systems, in the following form

$$\dot{x}_{2k+i} = -a_{ii}\, x_{2k+i} - \sum_{\substack{j=1 \\ j\neq i}}^{k} a_{ij}(e_j \cos\delta^s_{ij} - E_j \cos\delta_{ij}) \qquad (i=1,2,\ldots,k) \qquad (4.50)$$

where

$$a_{ii} = [X_{ii} + x_{di} - x'_{di}]/(T'_{oi}\, X_{ii})$$

$$a_{ij} = (x_{di} - x'_{di})/(T'_{oi}\, X_{ij}) \qquad (4.51)$$

Define a kxm matrix K^{21} as

$$K = [K_1 \quad K_2 \quad \ldots \quad K_{k-1}]$$

where

$$K_j = \begin{bmatrix} 0_{((j-1)x(k-j))} \\ 1_{(1x(k-j))} \\ -I_{((k-j)x(k-j))} \end{bmatrix} \qquad (j=1,2,\ldots,k-1) \qquad (4.52)$$

and

$$m = k(k-1)/2$$

The subscripts of the form (ixj) attached to the various submatrices here and elsewhere denote the dimensions of the respective submatrices (i and j indicating the number of rows and columns respectively). In (4.52) above $1_{(1x(k-j))}$ is a row matrix having all its elements equal to unity and $I_{((k-j)x(k-j))}$ is a unit matrix of order (k-j).

Now define a (m+k)-vector

$$\underline{\sigma} = \begin{bmatrix} \underline{\Sigma}_1 \\ \underline{\Sigma}_2 \end{bmatrix} \tag{4.53}$$

where the m-vector $\underline{\Sigma}_1$ and the k-vector $\underline{\Sigma}_2$ are defined by

$$\underline{\Sigma}_1 = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_m \end{bmatrix} = K^T \underline{X}_1 \; ; \quad \underline{\Sigma}_2 = \begin{bmatrix} \sigma_{m+1} \\ \sigma_{m+2} \\ \vdots \\ \sigma_{m+k} \end{bmatrix} = \underline{X}_3 \tag{4.54}$$

Also define a (m+k)-vector $\underline{f}(\underline{\sigma})$, respresenting the nonlinearities as

$$\underline{f}(\underline{\sigma}) = \begin{bmatrix} \underline{\emptyset}_1(\underline{\sigma}) \\ \underline{\emptyset}_2(\underline{\sigma}) \end{bmatrix} \tag{4.55}$$

where

$$\underline{\emptyset}_1(\underline{\sigma}) = \text{col}(f_1(\underline{\sigma}),\ f_2(\underline{\sigma}),\ \ldots,\ f_m(\underline{\sigma}))$$

$$\underline{\emptyset}_2(\underline{\sigma}) = \text{col}(f_{m+1}(\underline{\sigma}),\ f_{m+2}(\underline{\sigma}),\ \ldots,\ f_{m+k}(\underline{\sigma})) \tag{4.56}$$

The components of $\underline{\emptyset}_1(\underline{\sigma})$ are defined by

$$f_i(\underline{\sigma}) = (\sigma_{k+j}+e_j)(\sigma_{k+p}+e_p)Y_{jp}\sin(\sigma_i+\delta^s_{jp})-e_j e_p Y_{jp}\sin\delta^s_{jp}$$

$$(i=1,2,\ldots,m) \qquad (4.57)$$

where j and p are the indices of the components of $\underline{X}_1$ on which σ_i is dependent.

The k-vector $\underline{\emptyset}_2(\underline{\sigma})$ has components defined by

$$f_{m+i} = \sum_{\substack{j=1 \\ j\neq i}}^{k} a_{ij}[e_j\cos\delta^s_{ij} - (\sigma_{m+j}+e_j)\cos(\sigma_{ij}+\delta^s_{ij})]$$

$$(i=1,2,\ldots,k) \qquad (4.58)$$

where

$$\sigma_{ij} = \sigma_p \quad \text{if} \quad i < j$$

$$= -\sigma_p \quad \text{if} \quad i > j \qquad (4.59)$$

where σ_p is that component of the vector $\underline{\Sigma}_1$ which is dependent on components of $\underline{X}_1$ having indices i and j (i.e. $\sigma_p = x_i - x_j$). Using the various relationships given above, the system equations (4.46) and (4.50) can be cast in the following form

$$\dot{\underline{X}}_1 = \underline{X}_2$$

$$\dot{\underline{X}}_2 = -D\,M^{-1}\,\underline{X}_2 - M^{-1}\,K\,\underline{\emptyset}_1(\underline{\sigma})$$

$$\dot{\underline{X}}_3 = -A_o\,\underline{X}_3 - \underline{\emptyset}_2(\underline{\sigma}) \qquad (4.60)$$

where the kxk diagonal matrices M, D and A_0 are given by

$$M = \text{diag}(M_i) \; ; \quad D = \text{diag}(D_i) \; ; \quad A_0 = \text{diag}(a_{ii}) \qquad (4.61)$$

Now introducing k new variables denoted by a k-vector $\underline{\xi}$ and defined by

$$\underline{\xi} = D\,\underline{X}_1 + M\,\underline{X}_2 \qquad (4.62)$$

the system can be represented by the standard form

$$\begin{aligned} \dot{\underline{X}} &= F\,\underline{X} - G\,\underline{f}(\underline{\sigma}) \\ \dot{\underline{\xi}} &= -N\,\underline{f}(\underline{\sigma}) \\ \underline{\sigma} &= H^T\,\underline{X} + P\,\underline{\xi} \end{aligned} \qquad (4.63)$$

where $\underline{X} = \begin{bmatrix} \underline{X}_2 \\ \underline{X}_3 \end{bmatrix}$; F is a 2kx2k matrix; G is a (2kx(m+k)) matrix; N is a (kx(m+k)) matrix; H is a (2kx(m+k)) matrix and P is a ((m+k)xk) matrix (all real and constant matrices) and are given by

$$F = \begin{bmatrix} -DM^{-1} & 0_{(kxk)} \\ 0_{(kxk)} & -A_0 \end{bmatrix} \; ; \quad G = \begin{bmatrix} M^{-1}K & 0_{(kxk)} \\ 0_{(kxm)} & I_{(kxk)} \end{bmatrix} \; ;$$

$$N = [K \quad 0_{(kxk)}] \; ; \quad H^T = \begin{bmatrix} -K^T MD^{-1} & 0_{(mxk)} \\ 0_{(kxk)} & I_{(kxk)} \end{bmatrix} \; ;$$

$$P = \begin{bmatrix} K^T D^{-1} \\ \\ 0_{(kxk)} \end{bmatrix} \qquad (4.64)$$

Liapunov Function for the k-Machine System:

Using the method of integration by parts as in the two-machine and three-machine systems, we will get a Liapunov function of the form

$$V(\underline{X},\underline{\sigma}) = \sum_{i=1}^{k} \ell_{ii} x_{k+i}^2 + \sum_{i=1}^{k} \ell_{(k+i)(k+i)} x_{2k+i}^2 - \sum_{i=1}^{m} \alpha_i \, x_{2k+p} \, x_{2k+j} + \sum_{i=1}^{m} \int_0^{\sigma_i} f_i(\underline{\sigma}) d\sigma_i \tag{4.65}$$

with the time derivative

$$\dot{V} = -2 \sum_{i=1}^{k} \ell_{ii} \lambda_i x_{k+i}^2 - 2 \sum_{i=1}^{k} \ell_{(k+i)(k+i)} (\dot{x}_{2k+i})^2 / a_{ii} \tag{4.66}$$

where

$\ell_{ii} = M_i/2 \quad (i = 1,2,\ldots,k)$; $\alpha_i = Y_{jp} \cos\delta_{jp}^S$;

$$\ell_{k+i} = \frac{1}{2} \, \frac{(X_{ii} + x_{di} - x'_{di})}{X_{ii}(x_{di} - x'_{di})} \quad (i=1,2,\ldots,k) \tag{4.67}$$

where j and p are the column indices of the first and second nonzero elements respectively of the ith row of K^T. While $\dot{V}$ is seen clearly to be negative semidefinite, the positive definiteness of V can be guaranteed by imposing conditions similar to the ones given in Section 4.3, which in essence requires that the quadratic part in the V-function (4.65) should be positive definite. These conditions can be obtained by applying Sylvester's

criterion as before. It is reasonable to expect that in realistic systems, these conditions will be satisfied.

4.5 CONCLUSION

The theme of this chapter has been the incorporation of flux decay effect in a multimachine system, which has hitherto been investigated with the assumption of constant field flux linkages. The immediate effect of the addition of this factor is to render the problem more complicated. The methods that have been used successfully to derive Liapunov functions in the earlier chapters, could not be extended to this problem. A version of 'integration by parts' method has been found suitable to generate Liapunov functions for such systems. A two-machine system and a three-machine system have been formulated and appropriate Liapunov functions systematically derived. These results have been used to formulate the problem of k-machine system and to arrive at a V-function. It is interesting to note that, if the flux decay effect is neglected, the V-function derived here resembles the energy type functions derived in the literature[7,8,21].

The voltage regulator and governor dynamics can in principle, be incorporated into the model examined in this chapter, on similar lines adopted for the single-machine system of Chapter III. It is to be hoped that the results of this chapter will form the basis for detailed numerical experimentation.

CHAPTER V

POWER SYSTEMS WITH TRANSFER CONDUCTANCES

5.1 INTRODUCTION

This chapter is concerned with seeking Liapunov functions which can be systematically constructed for power systems including the effect of transfer conductances. Although most transient stability studies in the past have been conducted with the transfer conductances neglected, in real systems these are far from negligible, in particular due to the constant impedance loads that might be present at the buses. Therefore an accurate analysis may require the inclusion of the same. Inclusion of transfer conductances does not present any difficulty in the simulation method of stability analysis. However, when the stability investigation is made through Liapunov method, this additional detail introduces some difficulties. Although the number of nonlinearities - each of which satisfies individually the 'sector conditions' - becomes twice that for the system with transfer conductances neglected, one will expect that a matrix version of Kalman's construction procedure must be able to lead to a Liapunov function. Indeed it does succeed in

yielding a Liapunov function for a two-machine system. But as will be shown later in the chapter, the problems of systems with three or more machines are not amenable to the procedure as the matrix Popov frequency condition cannot be satisfied. This problem seems to have evaded a satisfactory solution in the earlier works also, as discussed briefly below.

Aylett[6] considered the inclusion of transfer conductances in his energy integral formulation. However, as pointed out by Williams[63] these energy integrals are not strictly Liapunov functions, although these functions can be used for stability studies. Moreover, Aylett neglects the transfer conductances in certain terms when they present difficulties in integration while retaining the same in certain other terms where they do not pose any computational difficulty. Ribbens-Pavella[28] recasts the problem as a set of first order differential equations in the k(k-1) dimensional phase space (where k is the number of machines) in order to achieve what he terms 'symmetry'. He then proposes a Liapunov function making assumptions analogous to those of Aylett. The procedures followed by both these authors are applicable only for uniformly damped systems. (In fact they neglect damping). Further there is lack of consistency since the transfer admittances

are assumed as purely susceptive in some terms while in some other terms involving the same pair of indices no such assumption is made. In the process, some terms with indefinite sign are ignored in $\dot{V}$ which is ultimately shown to be identically zero[17]. El-Abiad and Nagappan[7] have included transfer conductances in their Liapunov function but have rightly restricted its applicability to Lagrange stability which implies boundedness of trajectories. Lüders[25] proposes a Liapunov function for such systems, again with some approximations (neglecting the nonintegrable part of V). The resulting V-function however, is practically the same as the one he proposes for the system without transfer conductances (except for a constant multiplier ranging in value between 0.96 to 1.0, associated with some terms in V). Doubts have been raised whether the V-function derived by Lüders is not a 'first integral' instead of a Liapunov function[64].

It is therefore evident that the inclusion of transfer conductances is not straightforward. A two-machine system even with damping included, can however be handled. Pai and Murthy[23] have derived Liapunov functions for such a system using a generalization of Popov's criterion due to Moore and Anderson[35]. This problem will be investigated in this work through Kalman's construction procedure. The problem of

three-machine system will be formulated to exhibit the 'lack of symmetry' in the transfer function matrix, which precisely is the reason why the Popov frequency criterion is not satisfied, resulting in the failure of this particular line of attack.

5.2 TWO-MACHINE SYSTEM

Formulation of the Problem:

With the usual assumptions including constant field flux linkages and constant input and neglecting transient saliency the two-machine system with transfer conductances can be characterized by the following dynamic equations

$$
\begin{aligned}
M_1\ddot{\delta}_1 + D_1\dot{\delta}_1 &= P_{m1} - P_{e1} \\
M_2\ddot{\delta}_2 + D_2\dot{\delta}_2 &= P_{m2} - P_{e2} \qquad (5.1)
\end{aligned}
$$

The electrical powers P_{e1} and P_{e2} are given by

$$
\begin{aligned}
P_{e1} &= \frac{E_1^2}{Z_{11}} \sin \alpha_{11} + \frac{E_1E_2}{Z_{12}} \sin(\delta_{12} - \alpha_{12}) \\
P_{e2} &= \frac{E_2^2}{Z_{22}} \sin \alpha_{22} + \frac{E_1E_2}{Z_{12}} \sin(\delta_{21} - \alpha_{12}) \qquad (5.2)
\end{aligned}
$$

where now Z_{ii} and Z_{ij} are the driving point and transfer impedances respectively, corresponding to the transient performance of the synchronous machines (transient impedances). α_{ii} and α_{ij} are the compliments

of θ_{ii} and θ_{ij} respectively; θ_{ii} and θ_{ij} are the impedance angles of Z_{ii} and Z_{ij} respectively. Further $Z_{ij} = Z_{ji}$ and $\delta_{ij} = \delta_i - \delta_j$. Substituting (5.2) into (5.1) and simplifying, we have

$$\ddot{\delta}_1 = -\frac{D_1}{M_1}\dot{\delta}_1 - \frac{1}{M_1}\left[\frac{E_1E_2}{Z_{12}}\sin(\delta_{12}-\alpha_{12}) - \frac{E_1E_2}{Z_{12}}\sin(\delta_{12}^s - \alpha_{12})\right]$$

$$\ddot{\delta}_2 = -\frac{D_2}{M_2}\dot{\delta}_2 - \frac{1}{M_2}\left[\frac{E_1E_2}{Z_{12}}\sin(\delta_{21} - \alpha_{12}) - \frac{E_1E_2}{Z_{12}}\sin(\delta_{21}^s - \alpha_{12})\right] \qquad (5.3)$$

Note that in arriving at (5.3), the term $(P_{mi} - \frac{E_i^2}{Z_{ii}} \sin \alpha_{ii})$ has been replaced by the second term in the square bracket of (5.3), in terms of the post fault stable equilibrium points δ_i^s $(i=1,2)$.

Define

$$\begin{aligned} x_1 &= \delta_1 - \delta_1^s \\ x_2 &= \delta_2 - \delta_2^s \\ y_1 &= \dot{x}_1 = \dot{\delta}_1 \\ y_2 &= \dot{x}_2 = \dot{\delta}_2 \end{aligned} \qquad (5.4)$$

Define also

$$\begin{aligned} \sigma_1 &= x_1 - x_2 \\ \sigma_2 &= x_2 - x_1 \end{aligned} \qquad (5.5)$$

Let the nonlinearities be defined as

$$f_1(\sigma_1) = \frac{E_1E_2}{Z_{12}} [\sin(\sigma_1 + \delta^s_{12} - \alpha_{12}) - \sin(\delta^s_{12} - \alpha_{12})]$$

$$f_2(\sigma_2) = \frac{E_1E_2}{Z_{12}} [\sin(\sigma_2 + \delta^s_{21} - \alpha_{12}) - \sin(\delta^s_{21} - \alpha_{12})] \tag{5.6}$$

Using equations (5.4) to (5.6) and with $\lambda_1 = D_1/M_1$ and $\lambda_2 = D_2/M_2$, the system equations (5.3) can be cast in the form

$$\dot{x}_1 = y_1$$

$$\dot{x}_2 = y_2$$

$$\dot{y}_1 = -\lambda_1 y_1 - \frac{1}{M_1} f_1(\sigma_1)$$

$$\dot{y}_2 = -\lambda_2 y_2 - \frac{1}{M_2} f_2(\sigma_2) \tag{5.7}$$

with $\sigma_1 = x_1 - x_2$ and $\sigma_2 = x_2 - x_1$.

Define two new variables ξ_1 and ξ_2 as

$$\xi_1 = D_1x_1 + M_1y_1$$

$$\xi_2 = D_2x_2 + M_2y_2 \tag{5.8}$$

With these new variables, the two-machine system (5.7) can be recast in the standard form as

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 \\ 0 & -\lambda_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{bmatrix} \begin{bmatrix} f_1(\sigma_1) \\ f_2(\sigma_2) \end{bmatrix}$$

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_1(\sigma_1) \\ f_2(\sigma_2) \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} -\frac{M_1}{D_1} & \frac{M_2}{D_2} \\ \frac{M_1}{D_1} & -\frac{M_2}{D_2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{D_1} & \frac{-1}{D_2} \\ \frac{-1}{D_1} & \frac{1}{D_2} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \tag{5.9}$$

The transfer function matrix of the linear part of the above system can be derived to be

$$W(s) = \begin{bmatrix} \frac{1/M_1}{s(s+\lambda_1)} & -\frac{1/M_2}{s(s+\lambda_2)} \\ -\frac{1/M_1}{s(s+\lambda_1)} & \frac{1/M_2}{s(s+\lambda_2)} \end{bmatrix} \tag{5.10}$$

Liapunov Function for the Two-Machine System:

Now for the system (5.9) a Liapunov function is to be derived. This requires at the outset a choice of matrices R and Q such that the following matrix Popov frequency condition[38,47] is satisfied:

$$Z(j\omega) + Z^T(-j\omega) \geq 0 \tag{5.11}$$

where

$$Z(s) = [2RPN + Qs]W(s) \tag{5.12}$$

In (5.12), the matrix R must be symmetric and so chosen that the product RPN must be a diagonal matrix. In the

problem under study, the matrices P and N are such that, the above condition can be satisfied only by making the product RPN a zero matrix. This can be achieved by choosing R as a matrix with all elements equal. Two possibilities exist. Either a finite value can be chosen for the elements of R or R can be chosen as a zero matrix. It is verified that either choice results in the same Liapunov function because of the special nature of H^T and $\underline{\sigma}$. We may therefore, as well chose R as zero matrix. Thus choose

$$R = 0 \; ; \quad Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \tag{5.13}$$

Then the matrix Z(s) is seen to be

$$Z(s) = \begin{bmatrix} \dfrac{q_1/M_1}{s+\lambda_1} & -\dfrac{q_1/M_2}{s+\lambda_2} \\[2ex] -\dfrac{q_2/M_1}{s+\lambda_1} & \dfrac{q_2/M_2}{s+\lambda_2} \end{bmatrix} \tag{5.14}$$

The matrix Popov frequency condition then requires

$$Z(j\omega)+Z^T(-j\omega) =$$

$$\begin{bmatrix} \dfrac{2\lambda_1 q_1/M_1}{\lambda_1^2+\omega^2} & \dfrac{-(\frac{q_1\lambda_1}{M_2}+\frac{q_2\lambda_2}{M_1})+j\omega(\frac{q_1}{M_2}-\frac{q_2}{M_1})}{\omega^2+\lambda_1\lambda_2+j\omega(\lambda_1-\lambda_2)} \\[3ex] \dfrac{-(\frac{q_1\lambda_1}{M_2}+\frac{q_2\lambda_2}{M_1})-j\omega(\frac{q_1}{M_2}-\frac{q_2}{M_1})}{\omega^2+\lambda_1\lambda_2-j\omega(\lambda_1-\lambda_2)} & \dfrac{2\lambda_2 q_2/M_2}{\lambda_2^2+\omega^2} \end{bmatrix} \geq 0$$

(5.15)

This requires the restrictions

$$M_1q_1 = M_2q_2 \quad ; \qquad D_1q_1 = D_2q_2 \tag{5.16}$$

The equations (5.16) imply

$$\lambda_1 = \lambda_2 \tag{5.17}$$

Thus it is necessary to assume 'uniform damping' for the system if we are to succeed in arriving at a Liapunov function. Furthermore, one of the elements in the diagonal of Q can be chosen arbitrarily, while the other element is fixed by the relation (5.16).

With R and Q chosen and having established that the matrix Popov frequency condition is satisfied for the two-machine system with uniform damping, we proceed to construct a Liapunov function for the problem. The procedure to derive the L matrix is along the lines outlined in Chapter III through equations (3.25) - (3.28). However, because of the special nature of the matrices involved and the assumption of uniform damping, there is no need to do explicit spectral factorization involved in the procedure. The general technique of handling this problem in a k-machine system has been outlined by Pai[21]. A Liapunov function for the problem on hand is now derived as follows:

(i) A 2x2 matrix T(s) is to be found such that

$$\tfrac{1}{2}[Z(s) + Z^T(-s)] = T^T(-s)\ T(s)$$

With $\lambda_1 = \lambda_2 = \lambda$ and $q_2/M_1 = q_1/M_2$ from (5.16), it can be verified that

$$\tfrac{1}{2}[Z(s) + Z^T(-s)] = \frac{\lambda}{(s+\lambda)(-s+\lambda)} T^T T \qquad (5.18)$$

where

$$T^T T = \begin{bmatrix} q_1/M_1 & -q_1/M_2 \\ -q_1/M_2 & q_2/M_2 \end{bmatrix} \qquad (5.19)$$

Thus

$$T(s) = \frac{\sqrt{\lambda}}{s+\lambda} T \qquad (5.20)$$

Although in general T is to be determined by factorizing (5.19), in this case, it is possible to avoid the explicit factorization as can be seen in the sequel.

(ii) The matrix Γ is next determined from

$$\tfrac{1}{2}Q(H^TG + PN) + \tfrac{1}{2}(G^T H + N^T P^T)Q^T = \Gamma^T \Gamma$$

With the appropriate matrices in the above problem, it can be seen that

$$\Gamma = 0 \qquad (5.21)$$

(iii) The matrix U is solved in general from

$$T(s) - \Gamma = -U^T(sI - F)^{-1} G$$

which on substitution of the various matrices yields

$$\sqrt{\lambda}\ T = -U^T G \qquad (5.22)$$

Again without explicitly solving for U^T, we proceed directly to solve the matrix Liapunov equation.

(iv) The Liapunov matrix equation is given by

$$F^T L + LF = -U\,U^T \tag{5.23}$$

Premultiplying and post-multiplying (5.23) by G^T and G respectively, we have

$$G^T(F^T L + LF)G = -G^T U U^T G \tag{5.24}$$

With $F = -\lambda I$ and with (5.22), equation (5.24) reduces to

$$2G^T L G = T^T T \tag{5.25}$$

Assume the symmetric L matrix to be of the general form

$$L = \begin{bmatrix} \ell_{11} & \ell_{12} \\ \ell_{12} & \ell_{22} \end{bmatrix} \tag{5.26}$$

Substituting this in (5.25) and solving for the elements of the L matrix we get

$$\begin{aligned} &\ell_{11} = q_1 M_1/2 \;\; ; \;\; \ell_{12} = -q_1 M_1/2 \;; \\ &\ell_{22} = q_2 M_2/2 = q_1 M_1/2 \end{aligned} \tag{5.27}$$

The Liapunov function is therefore given by

$$V = \underline{y}^T L \underline{y} + \sum_{i=1}^{2} q_i \int_0^{\sigma_i} f_i(\sigma_i) d\sigma_i$$

which on substitution of the above matrix L and

some rearrangement, yields

$$V = \tfrac{1}{2}q_1M_1(y_1-y_2)^2 +$$

$$q_1 \int_0^{\sigma_1} \frac{E_1E_2}{Z_{12}}[\sin(\sigma_1+\delta_{12}^s-\alpha_{12})-\sin(\delta_{12}^s-\alpha_{12})]d\sigma_1 +$$

$$q_2 \int_0^{\sigma_2} \frac{E_1E_2}{Z_{12}}[\sin(\sigma_2+\delta_{21}^s-\alpha_{12})-\sin(\delta_{21}^s-\alpha_{12})]d\sigma_2 \tag{5.28}$$

with

$$\dot{V} = -q_1M_1\ \lambda(y_1-y_2)^2 \tag{5.29}$$

The V-function derived above can be used to study all aspects of transient stability of the two-machine system. This Liapunov function differs from those in the literature for two-machine systems in that the quadratic part does not represent the kinetic energy.

5.3 FORMULATION OF 3-MACHINE SYSTEM

The equations of motion of the three-machines can be written as

$$M_i\ddot{\delta}_i + D_i\dot{\delta}_i = P_{mi} - P_{ei} \quad (i=1,2,3) \tag{5.30}$$

The electrical power outputs of the machines are given by

$$P_{ei} = \frac{E_i^2}{Z_{ii}}\sin\alpha_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{3} \frac{E_iE_j}{Z_{ij}}\sin(\delta_{ij}-\alpha_{ij}) \quad (i=1,2,3) \tag{5.31}$$

Note here that $Z_{ij} = Z_{ji}$ and $\alpha_{ij} = \alpha_{ji}$. Substituting (5.31) into (5.30), we see on post fault steady state

$$P_{mi} - \frac{E_i^2}{Z_{ii}} \sin \alpha_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{3} \frac{E_i E_j}{Z_{ij}} \sin(\delta_{ij}^s - \alpha_{ij})$$

$$(i = 1,2,3) \qquad (5.32)$$

Using the above equations, the system (5.30) can be rewritten as

$$\ddot{\delta}_i = -\lambda_i \dot{\delta}_i - \frac{1}{M_i} \sum_{\substack{j=1 \\ j \neq i}}^{3} \frac{E_i E_j}{Z_{ij}} [\sin(\delta_{ij} - \alpha_{ij}) - \sin(\delta_{ij}^s - \alpha_{ij})]$$

$$(i=1,2,3) \qquad (5.33)$$

Define now,

$$x_1 = \delta_1 - \delta_1^s \; ; \quad x_2 = \delta_2 - \delta_2^s \; ; \quad x_3 = \delta_3 - \delta_3^s$$

$$y_1 = \dot{x}_1 \; ; \quad y_2 = \dot{x}_2 \; ; \quad y_3 = \dot{x}_3 \qquad (5.34)$$

Also define

$$\sigma_1 = x_1 - x_2 \; ; \quad \sigma_2 = x_1 - x_3 \; ; \quad \sigma_3 = x_2 - x_3 \; ;$$

$$\sigma_4 = x_2 - x_1 \; ; \quad \sigma_5 = x_3 - x_1 \; ; \quad \sigma_6 = x_3 - x_2 \qquad (5.35)$$

Let

$$K_1 = E_1 E_2 / Z_{12} \; ; \quad K_2 = E_1 E_3 / Z_{13} \; ; \quad K_3 = E_2 E_3 / Z_{23} \qquad (5.36)$$

Define the nonlinearities as

$$f_1(\sigma_1) = K_1[\sin(\sigma_1+\delta_{12}^s-\alpha_{12}) - \sin(\delta_{12}^s-\alpha_{12})]$$

$$f_2(\sigma_2) = K_2[\sin(\sigma_2+\delta_{13}^s-\alpha_{13}) - \sin(\delta_{13}^s-\alpha_{13})]$$

$$f_3(\sigma_3) = K_3[\sin(\sigma_3+\delta_{23}^s-\alpha_{23}) - \sin(\delta_{23}^s-\alpha_{23})]$$

$$f_4(\sigma_4) = K_1[\sin(\sigma_4+\delta_{21}^s-\alpha_{21}) - \sin(\delta_{21}^s-\alpha_{21})]$$

$$f_5(\sigma_5) = K_2[\sin(\sigma_5+\delta_{31}^s-\alpha_{31}) - \sin(\delta_{31}^s-\alpha_{31})]$$

$$f_6(\sigma_6) = K_3[\sin(\sigma_6+\delta_{32}^s-\alpha_{32}) - \sin(\delta_{32}^s-\alpha_{32})] \qquad (5.37)$$

Using equations (5.34) to (5.37), the system (5.33) can be written as

$$\dot{y}_1 = -\lambda_1 y_1 - \frac{1}{M_1} f_1(\sigma_1) - \frac{1}{M_1} f_2(\sigma_2)$$

$$\dot{y}_2 = -\lambda_2 y_2 - \frac{1}{M_2} f_3(\sigma_3) - \frac{1}{M_2} f_4(\sigma_4)$$

$$\dot{y}_3 = -\lambda_3 y_3 - \frac{1}{M_3} f_5(\sigma_5) - \frac{1}{M_3} f_6(\sigma_6) \qquad (5.38)$$

Define new variables

$$\xi_i = D_i x_i + M_i y_i \qquad (i=1,2,3) \qquad (5.39)$$

With these new variables defined in (5.39), the three-machine system can be recast in the standard form (1.8) with the various vectors given by

$$\underline{y} = \mathrm{col}(y_1, y_2, y_3)$$

$$\underline{f}(\underline{\sigma}) = \mathrm{col}(f_1(\sigma_1),\ f_2(\sigma_2), \ldots,\ f_6(\sigma_6))$$

$$\underline{\xi} = \mathrm{col}(\xi_1,\ \xi_2,\ \xi_3)$$

$$\underline{\sigma} = \mathrm{col}(\sigma_1,\ \sigma_2,\ \ldots,\ \sigma_6) \tag{5.40}$$

and the relevant matrices defined by

$$F = -\mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$$

$$G = \begin{bmatrix} 1/M_1 & 1/M_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/M_2 & 1/M_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/M_3 & 1/M_3 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} -\frac{M_1}{D_1} & \frac{M_2}{D_2} & 0 \\ -\frac{M_1}{D_1} & 0 & \frac{M_3}{D_3} \\ 0 & -\frac{M_2}{D_2} & \frac{M_3}{D_3} \\ \frac{M_1}{D_1} & -\frac{M_2}{D_2} & 0 \\ \frac{M_1}{D_1} & 0 & -\frac{M_3}{D_3} \\ 0 & \frac{M_2}{D_2} & -\frac{M_3}{D_3} \end{bmatrix} ; \quad P = \begin{bmatrix} \frac{1}{D_1} & -\frac{1}{D_2} & 0 \\ \frac{1}{D_1} & 0 & -\frac{1}{D_3} \\ 0 & \frac{1}{D_2} & -\frac{1}{D_3} \\ -\frac{1}{D_1} & \frac{1}{D_2} & 0 \\ -\frac{1}{D_1} & 0 & \frac{1}{D_3} \\ 0 & -\frac{1}{D_2} & \frac{1}{D_3} \end{bmatrix} \tag{5.41}$$

With the matrices defined as above, the transfer function matrix of the linear part of the above system given by

$$W(s) = H^T(sI - F)^{-1} G + \frac{1}{s} P N$$

is found to be

$$W(s)=\begin{bmatrix} w_1(s) & w_1(s) & -w_2(s) & -w_2(s) & 0 & 0 \\ w_1(s) & w_1(s) & 0 & 0 & -w_3(s) & -w_3(s) \\ 0 & 0 & w_2(s) & w_2(s) & -w_3(s) & -w_3(s) \\ -w_1(s) & -w_1(s) & w_2(s) & w_2(s) & 0 & 0 \\ -w_1(s) & -w_1(s) & 0 & 0 & w_3(s) & w_3(s) \\ 0 & 0 & -w_2(s) & -w_2(s) & w_3(s) & w_3(s) \end{bmatrix} \tag{5.42}$$

where

$$w_1(s) = \frac{1/M_1}{s(s+\lambda_1)} \quad ; \quad w_2(s) = \frac{1/M_2}{s(s+\lambda_2)} \quad ;$$

$$w_3(s) = \frac{1/M_3}{s(s+\lambda_3)} \tag{5.43}$$

Inspection of W(s) reveals that the transfer function matrix for the three-machine system does not have symmetric nonzero pattern. With this W(s), the matrix Popov frequency criterion is not satisfied even under the assumption of uniform damping. Hence the method fails in arriving at a Liapunov function. However,

because of the sufficiency nature of the Liapunov theorem, no conclusions can be made about the stability or instability of the system. A similar outcome is observed in systems in which more than three machines are present. It can be easily seen that if transfer conductances are neglected, then

$$f_i(\sigma_i) = -f_{i+k}(\sigma_{i+k}) \qquad (i = 1,2,\ldots,m)$$

and we need to define only m nonlinearities. The problem then reduces to the standard case which has been effectively solved in the literature.

5.4 CONCLUSION

In this chapter, the power system stability has been investigated taking into consideration the transfer conductances. In general, it is observed that the number of nonlinearities for such a system is twice that for the corresponding system with transfer conductances neglected. A two-machine system has been formulated and a Liapunov function derived through Kalman's construction procedure, although 'uniform damping' had to be assumed. Such an assumption,however is not unusual, as the inaccuracy involved is small. The three-machine system has been formulated on similar lines as the two-machine system, but a Liapunov function could not be constructed because of the

failure of the 'Popov frequency criterion' test. This problem calls for further work as no systematic and accurate method to construct a Liapunov function for systems with more machines than two, has yet been found.

CHAPTER VI

CONCLUSION

6.1 INTRODUCTION

Interest in the application of Liapunov method for the transient stability analysis has been gaining momentum in recent years. After an early breakthrough in this area by El-Abiad and Nagappan[7] followed by many others, there seemed to have followed a lean period with regard to the further advance in the application of this approach. In fact there appears to be some degree of pessimism on the part of the power industry regarding the utility of this method. This can be attributed to several factors. One of the striking drawbacks of this approach is the nature of the conservative results obtained. While this may be partly due to the sufficiency nature inherent in the Liapunov theorem itself, it is aggravated by the approximations made in modelling the system and the consequent imperfections in the Liapunov functions developed. In this context, the synchronous machine is the most vital element whose exact modelling is quite complex. Factors like damping saliency, flux decay, voltage regulator and governor dynamics are known to affect the stability property of a power system. It is therefore logical to expect, that a

Liapunov function derived for a model which includes all these details, will yield more accurate results. Inclusion of all these factors in the development of a Liapunov function however, is far from easy. Addition of greater details in the machine model results not only in a higher order system, but often in more complicated and greater number of nonlinearities. An important step in formulating these problems lies in properly defining the state variables and nonlinearities. Certain models result in 'multiargument' nonlinearities. In such situations it is not always easy to ensure that the nonlinearities shall, individually, satisfy the 'sector conditions' away from the origin, although at the origin they can be forced to vanish. One has to then rely more on the quadratic part of the Liapunov function dominating the contribution due to the integral terms even if the latter becomes negative in some region around the origin, so that the overall V-function still remains positive in a sufficiently large region around the origin, within which the stability region is contained. The sign definite property of the time derivative of the V-function also sometimes calls for some skilful manipulations and assumptions. Notwithstanding these drawbacks, the Liapunov approach could still play a complementary role in assessing transient stability by cutting down on the number of

detailed simulations that have to be carried out by the direct simulation method. This complementary role of Liapunov approach has not been well-recognised by the power industry. The recent studies of Williams[63], Ribbens-Pavella and others[65,66] on some real systems have amply demonstrated the utility and feasibility of the Liapunov approach. These researchers have applied the method for the transient stability study of fairly large systems and the results have been very satisfactory. With the incorporation of improved system models and the consequent improvement of Liapunov functions, the Liapunov approach might be more attractive and readily acceptable. The aim of this thesis has been therefore, to incorporate improved synchronous machine models while constructing Liapunov functions for both single and multimachine systems. In addition, the utility of the Liapunov approach in carrying out a comprehensive parameter analysis is also emphasized in the thesis.

6.2 REVIEW

With the above objective in view, saliency effect has been used in the machine model in the second chapter. Both single machine-infinite bus system and a two-machine system have been formulated

and appropriate Liapunov functions derived. A detailed parameter analysis of the single machine-infinite bus system demonstrates how the various parameters affect the transient stability properties. The results clearly indicate that saliency in general improves the stability property of the system. The extent of the improvement however, is also related in some way to other factors like the damping in the system, the initial load on the generator, the location of the fault and inertia of the machine. The increase in the critical clearing time is particularly pronounced with a lower initial load and higher damping coefficients. Similarly if the fault is away from the buses, saliency has greater influence on t_c than with a fault close to the buses. The parameter analysis in addition to demonstrating the effect of saliency on the stability behaviour, also illustrates the effects of other parameters on transient stability. Such a detailed analysis by the direct simulation method would have resulted in excessive computational effort. A similar study can be made for the two-machine system for which Liapunov functions have been derived for both uniform and nonuniform damping cases. One of the special features of the Liapunov functions that have been derived for both single-machine system as well as two-machine system is that the damping coefficient explicitly appears in the V-functions. It is

logical therefore to expect that these Liapunov functions are more accurate than those which do not contain the damping coefficient explicitly. The case of systems with more number of machines having saliency however, becomes highly complex and derivation of Liapunov function becomes difficult. This is because of the fact that the resulting system equations contain in addition to the nonlinear differential equations, a set of nonlinear algebraic equations containing the state variables implicitly. Adequate results are not available in stability theory at the present time to handle such problems. The extension of the method to a general k-machine system thus, seem to be impossible unless some approximations are made. One possible approximation that can be made is to assume the voltage E_{Qi} $(i = 1,2,\ldots,k)$ as equal to the voltage E_i $(i = 1,2,\ldots,k)$ (the latter being the voltage behind the direct axis transient reactance) while including the second harmonic term due to saliency effect in the electrical power output of the machine. There is however, some inconsistency in this approximation; for, by assuming E_{Qi} as equal to E_i, it is implied that $x_q = x_d'$ and in such an event, the second harmonic term in the electrical power will also disappear. The problem then reduces to one without saliency. Therefore this problem is still open for further investigation.

The study of single machine-infinite bus system has been continued further in the third chapter. As a prelude to a generalized model of a single-machine system, flux decay and voltage regulator action have been included in the modelling of the synchronous machine. The problem turns out to be one having multiargument type nonlinearity. A straight application of Kalman's construction procedure does not work for this case. The incorporation of results from the work of Desoer and Wu[40] in the Kalman's construction procedure, enables us to arrive at a Liapunov function. Using this Liapunov function an illustrative example has been studied. The effects of flux decay and the voltage regulator on the critical clearing time have been clearly shown. The results show that flux decay in general reduces the critical clearing time, or equivalently affects the transient stability adversely. On the other hand voltage regulator improves the stability property, by neutralizing the adverse effect of flux decay. The extent of neutralization and improvement of the critical clearing time however, is seen to vary with the main parameters K_V and T_V of the voltage regulator, the former being more effective than the latter. In fact, decreasing T_V (or equivalently increasing the speed of response of the voltage regulator), results in only a small increase

in the critical clearing time. An augmented Liapunov function has also been proposed, which gives slightly better results then the one referred to above. However, the choice of the coefficients of the additional terms in the quadratic part of the V-function requires some amount of search. It is not known whether an optimum pair of coefficients for one system will be good for another. This has to be established only after some experimentation with different systems. The remaining part of the third chapter has been devoted to the generalized model of the single-machine system, wherein saliency and governor dynamics have also been included in addition to the flux decay and voltage regulator. A V-function has been derived for this model using the modified Kalman's construction procedure.

With the success achieved with single-machine system, the next logical step is to extend the theory to multimachine systems. Since the saliency effect in a multimachine system made the problem more difficult to handle, only flux decay has been considered. The nature of the multiargument nonlinearities become more complex than in the single machine case. The procedure based on Popov's theorem were found to be not applicable. A version of integration by parts however, has been found

suitable. Using this technique, Liapunov functions have been systematically derived for the two-machine and three-machine systems. The results have been then extended to a general k-machine system. In this formulation, constraints on the parameters of the system in order to ensure the positive definiteness of the quadratic part of the Liapunov function, are found necessary. While this may appear to be restrictive, it is found by taking certain typical values for a large number of cases in three-machine systems, that these constraints are indeed satisfied. The results of this chapter should form a basis to include other details such as voltage regulator and governor dynamics. In fact the addition of governor dynamics (assumed linear) poses no problem. Augmenting the V-function by the addition of a square term of the state variable defined for the governor dynamics as in Section 3.3, Chapter III, for each machine, is all that is required. The coefficient associated with this term can be so chosen as to ensure the negative semidefinite property of $\dot{V}$. In principle, inclusion of voltage regulator must also be possible.

The fifth chapter has been devoted to the problem of power system with transfer conductances. Inclusion of transfer conductances in a Liapunov

function has been sucessfully demonstrated using Kalman's construction procedure for the two-machine system, although limited to the uniformly damped system. Extension to higher order systems has not been possible since for such systems the Popov frequency condition is not satisfied. This problem remains still very much open for further work. Even in the two machine system that has been solved, assumption of uniform damping is one limitation, although this is not too serious since the difference between the results for power systems with uniform and nonuniform damping is small.

6.3 AREAS FOR FURTHER WORK

Based on this research work, there are still some unsolved problems in this field and are briefly explained below:

1. Inclusion of saliency for a general k-machine system has to be investigated. As mentioned elsewhere, formulation of the general k-machine system results in a set of implicit nonlinear differential equations and nonlinear algebraic equations and presently no results are available in stability theory to investigate such a problem. Some judicious approximations have to be made if existing theoretical results are to be applicable.

(2) More exact representation of voltage regulator dynamics are desirable. The voltage regulator action is responsive to terminal voltage. But, if terminal voltage is to be brought into the picture in a detailed representation of voltage regulator (as opposed to a simple representation as in this thesis), one again encounters the problem as in (1) above, namely a set of nonlinear differential equations and nonlinear algebraic equations. This calls for some basic research in control theory.

(3) Inclusion of voltage regulator action and governor dynamics in the formulation of multimachine systems discussed in Chapter IV, has to be persued. Analytically this problem may sound formidable, but the success achieved in the case of single-machine system in Chapter III encourages one to extend the results to multimachine systems.

(4) Inclusion of transfer conductances for the general k-machine system ($k \geq 3$) can also be investigated further. This problem has eluded several researchers in the past. As shown in this thesis, Popov based methods are not successful. It is perhaps interesting to see if integration by parts method will yield some appropriate Liapunov function.

(5) In the Liapunov function constructed in this thesis, for systems involving multinonlinearities (both single-machine with flux decay and multimachine

systems) it has not been possible to include damping explicitly in the Liapunov functions. This is because of the inability to find a nonzero R in the multiplier (2 RPN + Qs) appearing in the matrix Popov frequency criterion. On a close examination, it was found that this difficulty has been due to nonsingular nature of PN and the consequent inability to select a symmetric R matrix such that RPN is diagonal. The singularity of PN seems to be inherent in the power system models that have been used for multimachine systems. On the other hand, in the formulation based on the theorem of Moore and Anderson[35], the multiplier is (A+Bs) with $A \geq 0$, $B \geq 0$, $(A+B) > 0$, and A and B being diagonal. Here A and B are not directly dependent on the system equations and it is possible to include damping parameter explicitly in the V-function through this approach[24]. The fact that the two procedures do not yield identical results have to be reconciled. A modification of multinonlinear M-K-Y lemma appears to be necessary to make the multiplier independent of the system equations. This is a problem for further research in nonlinear stability theory. Such a solution will pave the way to automatically include the damping constants in the V-function.

LIST OF REFERENCES

1. E.W. Kimbark, Power System Stability, Vol.I (Elements of Stability Calculations), John Wiley and Sons, New York, 1948.

2. S.B. Crary, Power System Stability, Vol.II (Transient Stability), John Wiley and Sons, New York, 1947.

3. D.N. Ewart and F.P. DeMello, 'A Digital Computer Program for the Automatic Determination of Dynamic Stability Limits', IEEE Trans. Power App. Syst., Vol. PAS-86, pp. 867-874, 1967.

4. N. Dharma Rao, 'A New Approach to the Transient Stability Problem', AIEE Trans. Power App. Syst., Vol. 81, pp. 186-192, 1962.

5. P.C. Magnusson, 'The Transient Energy Method of Calculating Stability', AIEE Trans. Power App. Syst., Vol.66, pp. 747-755, 1947.

6. P.D. Aylett, 'The Energy Integral Criterion of Transient Stability Limits of Power Systems', Proc. IEE, Vol.77, pp. 527-536, 1958.

7. A.H. El-Abiad and K. Nagappan, 'Transient Stability Regions of Multimachine Power Systems', IEEE Trans. Power App.Syst., Vol.PAS-85, pp.169-179, 1966.

8. G.E.Gless, 'The Direct Method of Liapunov Applied to Transient Power System Stability', IEEE Trans. Power App.Syst., Vol.PAS-85, pp. 159-168, 1966.

9. A.G.J. McFarlan, M.A. Laughton and G.F.Shannon, 'Generalized Nonlinear Network Analysis Applied to the Dynamic Behaviour of Synchronous Machine Systems', Int. J. Control, Vol.3, pp. 97-111, 1966.

10. F. Fallside and M.R. Patel, 'On the Application of Liapunov Method to Synchronous Machine Stability', Int. J. Control, Vol.4, pp. 501-513, 1966.

11. Y.N.Yu and K. Vongsuriya, 'Nonlinear Power System Stability Study by Liapunov Function and Zubov's Method', IEEE Trans. Power App. Syst., Vol. PAS-86, pp. 1480-1485, 1967.

12. M.H. Siddiqee, 'Transient Stability of an A.C. Generator by Liapunov's Direct Method', Int. J. Control, Vol.8, pp. 131-144, 1968.

13. N. Dharma Rao, 'Generalization of Liapunov Functions for the Transient Stability Problem', Trans. Eng. Inst., Canada, Vol.11, No.C-3, pp. I-XI, 1968.

14. J.L. Willems, 'Improved Liapunov Function for Transient Power System Stability', Proc. IEE, Vol. 115, pp. 1315-1317, 1968.

15. J.L. Willems, 'Optimum Liapunov Function and Stability Regions for Multimachine Power Systems', Proc. IEE, Vol. 117, pp. 573-577, 1970.

16. J.L. Willems and J.C.Willems, 'The Application of Liapunov Methods to the Computation of Transient Stability Regions for Multimachine Power Systems', IEEE Trans. Power App.Syst., Vol. PAS-89, pp.795-801, 1970.

17. J.L. Willems, 'Direct Methods for Transient Stability Studies in Power System Analysis', IEEE Trans. on Aut. Control, Vol. AC-16, pp. 332-341, 1971.

18. N. Dharma Rao, 'Routh-Hurwitz Condition and Liapunov Methods for the Transient Stability Problem', Proc. IEE, Vol.116, pp.539-547,1969.

19. R.D. Teichgraeber, F.W.Harris and G.L.Johnson, 'New Stability Measure for Multimachine Power Systems', IEEE Trans. Power App.Syst., Vol. PAS-89, pp. 233-239, 1970.

20. M.A. Pai, M. Anandamohan and J. Gopala Rao, 'Power System Transient Stability Regions using Popov's Method', IEEE Trans. Power App. Syst., Vol. PAS-89, pp. 788-794, 1970.

21. M.A.Pai, 'Transient Stability Studies in Power Systems using Liapunov-Popov Approach', Presented at the 5th IFAC Congress, Paris, June 12-17, 1972.

22. M.A. Pai and M. Anandamohan, 'Transient Stability of Multimachine Systems by Popov's Method', IEEE Paper No. 70-CP-667, PWR, Summer Power Meeting, Los Angeles, July 1970.

23. M.A. Pai and P.G. Murthy, 'On the Liapunov Functions for Power Systems with Transfer Conductances', IEEE Trans. Aut.Control, Vol. AC-18, pp. 181-183, 1973.

24. M.A. Pai and P.G. Murthy, 'New Liapunov Functions for Power Systems Based on Minimal Realizations', To appear in Int. J. Control.

25. G.A. Lüders, 'Transient Stability of Multimachine Power Systems via the Direct Method of Liapunov', IEEE Trans. Power App. Syst., Vol. PAS-90, pp. 23-36, 1971.

26. A.K.DeSarkar and N. Dharma Rao, 'Zubov's Method and Transient Stability Problems of Power Systems', Proc. IEE, Vol.118, pp. 1035-1040, 1971.

27. M. Mansour, 'Stability Analysis and Control of Power Systems', Real Time Control of Electric Power Systems (Book), Ed: Edmund Handschin, Elsevier Publishing Company, Amsterdam, 1972.

28. M. Ribbens-Pavella, 'Transient Stability of Multimachine Power Systems by Liapunov's Direct Method', IEEE Paper No. 71-CP-17-PWR, Winter Power Meeting, New York, Jan/Feb. 1971.

29. D.G.Schultz and J.E. Gibson, 'The Variable Gradient Method for Generating Liapunov Functions', AIEE Trans. Power App. Syst., Vol. 81, pp.203-210, 1962.

30. R. Reiss and G. Geiss, 'The Construction of Liapunov Functions', IEEE Trans. Aut. Control, Vol. AC-8, pp. 382-383, 1963.

31. E.F. Infante and L.G. Clark, 'A Method for the Determination of the Domain of Stability of Second Order Nonlinear Autonomous Systems', Journal of App. Mech., Vol. 31m Trans. A.S.M.E., Vol. 86, Series E,pp. 315-320, 1964.

32. V.I.Zubov, The Methods of A.M. Liapunov and Their Applications (in Russian, Leningrad University Publishing House, 1957), U.S.Atomic Energy Commission translation AEC-Tr-4439, 1961.

33. V.M.Popov, 'Absolute Stability of Nonlinear Systems of Automatic Control', Automation and Remote Control, Vol.22, pp. 857-875, 1962.

34. R.E. Kalman, 'Liapunov Functions for the Problem of Lur'e in Automatic Control', Proc. Nat.Acad. Sci. (U.S.A.), Vol.49, No.2, pp.201-205, 1963.

35. J.B. Moore and B.D.O.Anderson, 'A Generalization of the Popov Criterion', Journal Franklin Inst., Vol. 285, pp. 488-492, 1968.

36. M.A. Aizerman and F.R. Gantmacher, Absolute Stability of Regulator Systems, Holden-Day, San Francisco, 1964.

37. S.Lefschetz, Stability of Nonlinear Control Systems, Academic Press, New York, 1965.

38. K.S. Narendra and R.M. Goldwyn, 'Existence of Quadratic Type Liapunov Functions for a Class of Nonlinear Systems', Int.J.Engg.Sci., Vol.2, pp. 367-377, 1964.

39. K.S. Narendra and C.P.Neuman, 'Stability of Continuous Time Dynamical Systems with m-feedback Nonlinearities', AIAA Journal, Vol.5, No.11, pp. 2021-2027, 1967.

40. C.A.Desoer and M.Y.Wu, 'Stability of a Nonlinear Time-Invariant Feedback System Under Almost Constant Inputs', Automatica, Vol.5, pp.231-233, 1969.

41. E.W. Kimbark, Power System Stability, Vol.III (Synchronous Machines), John Wiley and Sons, New York, 1956.

42. A. Johannesen and J.A. Harle, 'Limiting Curves for Transient Stability (A Digital Program for Evaluating Operating Guides for Synchronous Generators Connected to Large Systems)', IEEE Trans. Power App.Syst., Vol.PAS-80, pp.768-774, 1961.

43. J.A. Walker and N.H. McClamroch, 'Finite Regions of Attraction for the Problem of Lur'e', Int.J. Control, Vol.6, No.4, pp. 331-336, 1967.

44. S.B. Crary, Power System Stability, Vol.I (Steady State Stability), John Wiley and Sons, New York, 1945.

45. V.A. Yacubovich, 'The Solution of Certain Matrix Inequalities in Automatic Control Theory', Dokl. Akad. Nauk. U.S.S.R. 143, pp.1304-1307, 1962.

46. J.P. LaSalle, 'Complete Stability of a Nonlinear Control System', Proc. Nat.Acad.Sci. (U.S.A.), Vol.48, pp. 600-603, 1962.

47. M. Anandamohan, 'Power System Transient Stability by Methods of Liapunov and Popov', Ph.D. Thesis, Indian Institute of Technology, Kanpur (India), June 1970.

48. K.R. Meyer, 'Liapunov Functions for the Problem of Lur'e', Proc. Nat.Acad.Sci. (U.S.A.), Vol.53, No.3, pp. 501-503, 1965.

49. M.A. Pai and Viswanatha Rai, 'Parameter Analysis in Transient Stability of Power Systems Including Saliency Effects', IEEE Paper No. C 73 079-1, Winter Power Meeting, New York, 1973.

50. M.A. Pai, Vishwanatha Rai and P.G. Murthy, 'Recent Advances in the Application of Liapunov Stability Theory to Power Systems', Presented at the International Conference on Systems and Control, P.S.G. College of Technology, Coimbatore (India), 1973.

51. P.P. Gupta and M.W. Humphrey Davies, 'Digital Computers in Power System Analysis', Proc. IEE, Part A, Vol. 108, pp. 383-404, 1961.

52. F.P. DeMello and C. Concordia, 'Concepts of Synchronous Machine Stability as Affected by Excitation Control', IEEE Trans. Power App.Systems, Vol. PAS-88, pp. 316-327, 1969.

53. M.S. Dyrkacz, C.C. Young, F.J. Maginnis, 'A Digital Transient Stability Program Including the Effects of Regulator, Exciter and Governor Response', AIEE Trans. Pt.III, Vol. PAS-88, pp. 1245-1254, 1961.

54. C. Concordia, Synchronous Machines; Theory and Performance, John Wiley and Sons, New York, 1951.

55. M.A. Pai and Vishwanatha Rai, 'Liapunov-Popov Stability Analysis of Synchronous Machine with Flux Decay and Voltage Regulator', To appear in Int. J. Control.

56. D.C. Youla, 'On the Factorization of Rational Matrices', IRE Trans. Inf. Th., Vol. IT-7, pp. 172-189, 1961.

57. R.W. Brockett, 'Spectral Factorization of Rational Matrices', Proc. Int. Conf. on Microwaves, Circuit Theory and Inf. Theory, Part 3, (Inst. of Elec. Communication Engineers of Japan), 1964.

58. R.J. Kavanagh, 'Spectral Matrix Factorization', Electronics Letters, Vol.4, No.24, pp. 527-528, 1968.

59. M.C. Davis, 'Factoring the Spectral Matrix', IEEE Trans. Aut.Control, Vol.AC-8, pp.296-305, 1963.

60. M.G. Strintzis, 'A Solution to the Matrix Factorization Problem', IEEE Trans. Inf. Theory, Vol. IT-18, No.2, 1972.

61. G.W. Stagg and A.H. El-Abiad, Computer Methods in Power System Analysis, McGraw-Hill Book Company, New York, 1968.

62. C.C. Young, 'Equipment and System Modelling for Large Scale Stability Studies', IEEE Trans. Power App. Syst., Vol. PAS-91, pp. 99-109, 1972.

63. H.F. Williams, S.A. Louie and G.W. Bills, 'Feasibility of Liapunov Functions for the Stability Analysis of Electric Power Systems Having upto 60 Generators', IEEE Trans. Power App. Syste., Vol. PAS-91, pp. 1145-1159, 1972.

64. H.F. Williams, and G.W. Bills, Discussion on 'Transient Stability of Multimachine Power Systems via the Direct Method of Liapunov', IEEE Trans. Power App. Syst., Vol.PAS-90, pp. 23-36, 1971.

65. M. Ribbens-Pavella and J.L.Horward, 'Practical Considerations on the Transient Stability Power System Studies by Liapunov's Direct Method', Revue E - vol VII - No.7, pp.167-179, 1973.

66. .. Calvaer, J.L. Horward and M. Ribbens-Pavella, 'Relative Transient Stability Power Systems Margin Index', IEEE Paper No. C 73-474-4, Summer Power Meeting, 1973.

67. Joseph LaSalle and Solomon Lefschetz, Stability by Liapunov's Direct Method with Applications, Academic Press, New York, 1961.

68. J.P. LaSalle, 'Some Extensions of Liapunov's Second Method', IRE Trans. Circuit Theory, Vol.CT-7, pp. 520-527, 1960.

APPENDIX A

LIAPUNOV STABILITY

Consider an autonomous system[37,67]

$$\dot{\underline{x}} = \underline{X}(\underline{x}) \; ; \quad \underline{X}(\underline{0}) = \underline{0} \tag{A.1}$$

Let S_R denote the spherical region $\| \underline{x} \| < R$ and H_R the boundary sphere of the region. Assume that the system (A.1) is of class C^1 in a certain region S_ρ.

Liapunov defines the origin as:

'Stable' for (A.1) whenever given any $0 < \varepsilon < \rho$ there corresponds to it a $0 < \eta(\varepsilon) \leq \varepsilon$ such that if $\underline{x}(t)$ is a solution whose initial position $\underline{x}_o = \underline{x}(\underline{0})$ lies in S_η then $\underline{x}(t)$ lies in S_ε ever after;

'Asymptotically stable' whenever the origin is stable, and furthermore for some ε every solution $\underline{x}(t)$ as above $\to 0$ as $t \to +\infty$;

'Unstable' whenever given any $0 < \varepsilon < \rho$ and no matter what $0 < \eta < \varepsilon$, there is always an $\underline{x}(t)$ as above reaching H_ε at some $t > 0$.

I. Stability Theorem: Whenever for some S_ρ there exists a positive definite function $V(\underline{x})$ whose derivative $\dot{V}$ along the paths of (A.1) is negative semidefinite in S_ρ then the origin is stable.

II. Asymptotic Stability Theorem: Whenever $\dot{V}$ is actually negative definite in S_ρ, the origin is asymptotically stable.

III. The Barbashin-Krassovskii complement to the Asymptotic Stability Theorem: In the theorem II above let

(a) $S_\rho = S_\infty$;

(b) $V(\underline{x}) \to \infty$ with $\|\underline{x}\| \to \infty$

Then the system is asymptotically stable in the large or completely stable.

IV. Theorem of LaSalle[68]: Let $V(\underline{x})$ be a scalar function with continuous first partials satisfying

1) $V(\underline{x}) > 0$ for all $x \neq 0$; $V(\underline{0}) = 0$.

2) $\dot{V}(\underline{x}) \leq 0$ for all x

3) $V(\underline{x}) \to \infty$ as $\|\underline{x}\| \to \infty$

If $\dot{V}$ is not identically zero along any solution other than the origin, then the system (A.1) is completely stable.

Remark: In theorem IV the region $S_\rho = S_\infty$. If instead we have $\rho < \infty$, then the theorem is valid in the finite region S_ρ, which is termed as the region of attraction. Indeed power systems do belong to this category having only finite stability domain. It is this version of the theorem on which the results of the thesis have been based.

APPENDIX B

PROOF OF THE THEOREM IN SECTION 3.2.2

Assume the Liapunov function

$$V(\underline{y},\underline{\sigma}) = \underline{y}^T L \underline{y} + (\underline{\sigma} - H^T \underline{y})^T R(\underline{\sigma} - H^T \underline{y}) + V_1(\underline{\sigma}) \qquad (B.1)$$

The time derivative of V along the system equations can be shown to be

$$\dot{V} = \underline{y}^T(F^T L + LF)\,\underline{y} - 2\underline{y}^T(LG - HR\underline{\rho}\underline{d}^T - \tfrac{1}{2}F^T HQ^T)\underline{f}(\underline{\sigma})$$

$$-\underline{f}^T(\underline{\sigma})\,[\tfrac{1}{2}Q(H^T G + \underline{\rho}\,\underline{d}^T) + \tfrac{1}{2}(G^T H + \underline{d}\,\underline{\rho}^T)Q^T]\underline{f}(\underline{\sigma})$$

$$-2\underline{\sigma}^T R\,\underline{\rho}\,\underline{d}^T\underline{f}(\underline{\sigma}) \qquad (B.2)$$

Define

$$F^T L + LF = -\,\theta \qquad (B.3)$$

$$L\,G - H\,R\,\underline{\rho}\,\underline{d}^T - \tfrac{1}{2}F^T\,H\,Q^T = \mu \qquad (B.4)$$

$$\tfrac{1}{2}Q(H^T G + \underline{\rho}\,\underline{d}^T) + \tfrac{1}{2}(G^T H + \underline{d}\,\underline{\rho}^T)Q^T = \kappa \qquad (B.5)$$

With the above defined quantities

$$\dot{V} = -[\underline{y}^T\theta\,\underline{y} + 2\,\underline{y}^T\mu\,\underline{f}(\underline{\sigma}) + \underline{f}^T(\underline{\sigma})\kappa\,\underline{f}(\underline{\sigma})]$$

$$-2\,\underline{\sigma}^T R\,\underline{\rho}\,\underline{d}^T\,\underline{f}(\underline{\sigma}) \qquad (B.6)$$

Let the quantity in the square brackets be S. Together with the assumption a2, it is required that $S \geq 0$ to make $\dot{V} \leq 0$. Further it is necessary[38] that $\kappa \geq 0$.

It is thus possible to write

$$\kappa = \Gamma^T \Gamma \tag{B.7}$$

where Γ is a (mxm) matrix. Substituting for κ and with $\Gamma^T \Gamma > 0$, S can be written as

$$S = \| \Gamma \underline{f}(\underline{\sigma}) + U^T \underline{y} \|^2 - \underline{y}^T(\theta - U U^T) \underline{y} \tag{B.8}$$

where

$$U \Gamma = \mu \tag{B.9}$$

Define

$$\theta_1 = \theta - U U^T \tag{B.10}$$

To ensure $S \geq 0$, it is now necessary and sufficient that $\theta_1 \geq 0$. Thus if there exist a positive semi-definite matrix θ_1, a symmetric positive definite matrix L and a constant matrix U such that $U \Gamma = \mu$ and

$$\text{(a)} \quad F^T L + L F = -(\theta_1 + U U^T)$$

$$\text{(b)} \quad L G - \tfrac{1}{2}F^T H Q^T - H R \underline{\rho} \ \underline{d}^T = U\Gamma \tag{B.11}$$

then $S \geq 0$. The two equations in (B.11) are the so-called Lur'e's resolving prelimit equations for the multinonlinear case. If we take $\Gamma^T \Gamma = 0$, S can be made positive semidefinite in $\underline{\mathbf{y}}$ only by setting $\mu = 0$ and S would then become

$$S = \underline{y}^T \theta \underline{y} \tag{B.12}$$

The corresponding Lur'e's resolving equations are merely a particular case of (B.11). Hence (B.11) covers, in general, the case $\Gamma^T \Gamma \geq 0$. From (B.11) the Lur'e's resolving limit equations are obtained by letting $\theta_1 = 0$.

Now consider the matrix version of Kalman-Yacubovich lemma[38,39]:

Let Γ be a real (mxm) matrix, and F, a real ((n-1)x(n-1)) stable matrix. Let G and K be two ((n-1)xm) matrices. If

$$J(s) \triangleq K^T (sI-F)^{-1} G \qquad (B.13)$$

the frequency condition

$$\Gamma^T\Gamma + J(j\omega) + J^T(-j\omega) \geq 0 \quad \text{for all real } \omega \qquad (B.14)$$

is necessary and sufficient condition for the existence of a ((n-1) x (n-1)) real symmetric matrix L > 0 and a ((n-1)xm) real matrix U such that

$$\begin{aligned} &(a)\ F^T L + L F = -U U^T \\ &(b)\ L G - K = U \Gamma. \end{aligned} \qquad (B.15)$$

Comparing (B.15) with the limit equations corresponding to (B.11) (i.e. with $\theta_1 = 0$), the matrices F, G, U and Γ in both sets of equations are identical and furthermore, we equate

$$K = \tfrac{1}{2} F^T H Q^T + H R \underline{\rho}\ \underline{d}^T \qquad (B.16)$$

With these equivalences, (B.13) becomes after some algebraic manipulations,

$$J(s) = -\tfrac{1}{2}Q\, H^T G + (R\underline{\rho}\,\underline{d}^T + \tfrac{1}{2}Qs)H^T(sI-F)^{-1}G \qquad (B.17)$$

Using (B.5), (B.7) and (B.17), the left-hand side of the frequency condition (B.14) in the K-Y lemma can be shown to be

$$(R\underline{\rho}\,\underline{d}^T + \tfrac{1}{2}j\omega Q)H^T(j\omega I-F)^{-1}G + \tfrac{1}{2}Q\underline{\rho}\,\underline{d}^T +$$

$$[(R\underline{\rho}\,\underline{d}^T - \tfrac{1}{2}j\omega Q)H^T(-j\omega I-F)^{-1}G + \tfrac{1}{2}Q\underline{\rho}\,\underline{d}^T]^T$$

$$= \tfrac{1}{2}[(2R\underline{\rho}\,\underline{d}^T + j\omega Q)\{H^T(j\omega I-F)^{-1}G + \frac{\underline{\rho}\,\underline{d}^T}{j\omega}\}] - \frac{2R\underline{\rho}\,\underline{d}^T\underline{\rho}\,\underline{d}^T}{j\omega}$$

$$+ \tfrac{1}{2}[(2R\underline{\rho}\,\underline{d}^T - j\omega Q)\{H^T(-j\omega I-F)^{-1}G - \frac{\underline{\rho}\,\underline{d}^T}{j\omega}\}]^T$$

$$+ [(\frac{2R\underline{\rho}\,\underline{d}^T\underline{\rho}\,\underline{d}^T}{j\omega})]^T$$

$$= \tfrac{1}{2}[Z(j\omega) + Z^T(-j\omega)]$$

(since $R\underline{\rho}\,\underline{d}^T\,\underline{\rho}\,\underline{d}^T$ is seen to be symmetric).

Thus the frequency conditions (B.14) and (3.22) are equivalent. This completes the proof.

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(ii) 'Recent Advances in the Application of Liapunov Stability Theory to Power Systems' (with M.A. Pai and P.G. Murthy), Presented at the International Conference on Systems and Control, P.S.G. College of Technology, Coimbatore (India), 1973.

(iii) 'Liapunov-Popov Stability Analysis of Synchronous Machine with Flux Decay and Voltage Regulator' (with M.A. Pai), To appear in Int. J. Control.